1. Introduction

Ship hydrodynamics started with the assumption of thin ships. It was as early as the end of last century that Michell (1898) developed the theory of thin ships in uniform motion in still water, and derived the well-known formula for the wave resistance. It has been proved later by Peters and Stoker (1957) that this theory is a correct solution of first order in the perturbation expansion of the exact boundary-value problem. Attempts of the utilization of this theory in the practical field of shipbuilding have appeared since Weinblum (1930) and Wigley (1927) introduced Michell's theory to the naval architects community.

![Graph](image)

Fig. 1. Comparison of Michell resistance with measured wave-pattern resistance (Tsutsumi)

In spite of considerable success in the design problem where the thin-ship formulation is utilized to determine small-resistance hull-forms, thin-ship theory is quite unsatisfactory for predicting wave resistance. Fig. 1 shows a comparison between wave resistance calculated by Michell's formula and that determined from wave-pattern analysis of models of different width, Tsutsumi et al (1975). Good agreement is observed only in the narrowest model of beam to length ratio 1/15, while agreement is very poor for practical dimensions. There have been several attempts to obtain higher-order approximations. However, a few examples of computation of the second-order approximation seemingly have not been able to show any perceivable improvement, Eggers (1966).

Thin-ship theory assumes that the beam of the ship is much smaller than its length as well as its draft. This condition implies that the transverse section of the ship is like a thin wedge.
and is not compatible with the condition of actual hull forms. It is obvious that such a defect becomes serious when thin-ship theory is applied to vertical motions of a ship.

Strip theory has been widely employed in ship-motion studies, for the purpose of predicting hydrodynamic forces and moments acting on oscillating ships, since Weinblum and St. Denis (1950) introduced this theory as a method of predicting ship motions in a seaway. It is recognized that fairly good agreement is obtained by strip theory in comparison with measured data, Gerritsma and Beukelman (1964), and it is generally accepted that strip theory provides a very useful tool for assessing ship motions in a seaway. The basic premise of strip theory is that the fluid motion in a plane perpendicular to the longitudinal axis of the ship is identical with the two-dimensional fluid motion around a cylinder of infinite length with the same cross section as that of the ship. Therefore the three-dimensionality of the fluid motion is completely ignored. A defect caused by this simplification appears in the low-frequency range. For instance, strip theory predicts an infinite added mass at zero frequency. Moreover, it is difficult to account for the effect of forward speed in a rational way. In strip theory, the effect of forward speed appears only in the boundary condition on the hull surface, but it does not represent the substantial change in the flow field due to the presence of forward speed. Although the basis of strip theory is rather intuitive, an important implication is the condition that the dimension of each cross section is much smaller than the axial length. This is equivalent to the assumption of slender-body theory.

The concept of slender-body theory was founded by Munk (1920) in his work on aerodynamics of airships. Jones (1946) extended the same approximation to the lifting surface of small aspect ratio. Adams and Sears (1953) presented an excellent survey on the slender-body theory. These works employed a direct expansion of the kernel function by the slenderness ratio of the body in the slender-body formulation. Ward (1949) suggested another method of derivation. Instead of the direct expansion, the slender-body expansion was applied to the Laplace or Fourier transform of the solution.

While slender-body theory has shown much success in aeronautical sciences for a long time, its application in ship hydrodynamics came much later. The significance of applying slender-body theory in ship hydrodynamics seems to be attributed to the following facts:
1. Deviation of wave resistance predicted by thin ship theory from measured data.
2. Higher-order thin-ship theory – not promising
3. Neumann-Kelvin approximation – inconsistent with contradiction
4. Strip theory has no theoretical basis.
5. Prediction by strip theory – not applicable at low frequency
6. Strip theory can not duly account for effects of forward speed and three-dimensionality.
7. C.F.D. – not satisfactory in reliability and practical feasibility

There have been several precursory ideas in connection with applying slender-body theory to ship problems. One may observe a sprout of this concept in Havelock's (1963) approximation for the wave resistance in high speed. Cummins (1956) proposed the application of slender-body theory to wave-resistance computation in his unpublished thesis, but there was no further development thereafter in his theory. The first application of slender-body theory to ship problems is attributed to Korvin-Kroukovsky (1955) in his ship-motion research, but the free surface was not taken into account in his formulation. Grim's (1960) analysis on the three-dimensional effect in strip theory for oscillating ships was based on an idea similar to slender-ship theory which has been developed later, though not intentionally. Another example is the theory of slender planing surfaces which was discussed by Tulin (1957) and Maruo (1961).

The theory of slender ships has been opened by three papers published in 1962. Vossers (1962) was the first to formulate the fluid motion generated by an oscillating slender ship with forward speed. He discussed the wave resistance in steady forward motion as well. Ursell (1962) discussed an oscillating slender body at zero forward speed. Maruo (1962) investi-
gated the wave resistance of a slender ship. Tuck (1963) developed his theory of slender ships in steady forward motion, making use of the systematic expansion of singular perturbation. Joosen (1963) examined Vossers theory and showed some numerical results. The original form of the slender-ship theory of this kind has been developed further by Newman (1964) for oscillating ships with zero forward speed. The oscillating slender ship with finite forward speed was discussed by Newman and Tuck (1964), Maruo (1966) and Ogilvie (1970). However, computational results by the original slender-ship formulation were rather disappointing, showing great deviations from measured data. Ogilvie and Tuck (1969) developed a formulation for oscillations of a slender ship at high frequency. If the forward speed is not present, the result is reduced to strip theory. The effect of finite forward speed appears in the boundary conditions both on the hull surface and on the free surface in a little higher order. This result is regarded as the rationalization of strip theory which may originally have been an intuitive approach.

The practical application of this result to assessing ship motions in a seaway was proposed by Solvesen, Tuck and Faltinsen (1970). Although strip theory seems to have a rational basis for the radiation problem at high frequency oscillations, it is probably not so for the problem of diffraction of ambient waves by a slender ship. The diffraction problem in short waves was formulated by Faltinsen (1972) and Maruo and Sasaki (1974). There are further investigations with respect to short-wave diffraction such as by Troesch (1976) and Mei (1978). The latter discussed the non-linear deformation of reflected waves by a slender ship. Since the high-frequency theory of the radiation problem is regarded as a refinement of the ship theory, it does not seem to be a genuine three-dimensional slender-ship theory, while the three-dimensional theory is substantial for problems of low frequency oscillations and steady forward motion.

Although the original slender-ship theory is a true three-dimensional formulation, it does not provide any satisfactory result when compared with actual phenomena as already mentioned. Maruo (1974) pointed out that the frequency of oscillations and the speed of advance have serious effects on the validity of the perturbation expansion. There are several attempts to find out a formulation which is valid throughout a wider frequency range. The interpolation theory by Maruo (1970) and the unified theory by Newman (1978) are typical approaches of this kind. Although derived by different concepts, the formulation resulting from both theories is nearly equivalent. These theories are developed originally for the radiation problem without forward speed, and their computational results show plausible agreement with measured data, Maruo and Tokura (1976) and Newman (1980). Later the extension of the theories to the case of oscillations with forward speed has been considered. In order to improve the theory of a slender ship in steady forward motion, Ogilvie (1972) proposed the bow near-field hypothesis. The original slender-ship theory takes the flow field near the ship as that of the double body in an unbounded fluid, but Ogilvie pointed out that the wave generation should be taken into account in the flow field near the bow. Applications of this concept were discussed by his followers, Adachi (1974), Reed (1975), Hirata (1972). Noblesse (1981) considered another approximation starting from the Neumann-Kelvin approximation, but he did not derive his theory from perturbation analysis. Since the slender-body theory is regarded as a perturbation solution of an originally non-linear boundary-value problem, this paper intends to discuss the development of slender-ship theory in the light of rational perturbation analysis.

2. Formal Perturbation Expansion

As basic assumption, the fluid is regarded as inviscid and incompressible. Then the fluid motion around a non-lifting body starting from rest is irrotational, and the flow field is specified by the velocity potential \( \Phi \), which satisfies the Laplace equation

\[
[L] \quad \nabla^2 \Phi = 0 \tag{1}
\]

in the fluid domain. Take Cartesian coordinates with axes \( x \) and \( y \) in the undisturbed free
surface and axis $z$ vertically upward. Let us consider a uniform flow of velocity $U$ in the direction of positive $x$, and a ship floating in the flow with its average position fixed in space (Fig. 2).

Fig. 2. Coordinate system

The ship is oscillating with small amplitude around its average position. Infinite depth of water is assumed. If $\vec{v}$ is the velocity of a point on the hull surface induced by the oscillation, the boundary condition on the hull surface is expressed by

$$[H] \quad \frac{\partial \Phi}{\partial n} = \vec{v} \vec{n}$$

where $n$ or $\vec{n}$ is the unit outward normal on the hull surface. The fluid motion satisfies the boundary conditions on the free surface, which consist of the kinematic condition and the dynamic condition. If the free surface is expressed by the equation

$$z = \zeta,$$

the boundary conditions on it are

$$[K] \quad \text{kinematic} \quad \zeta_t + \Phi_x \zeta_x + \Phi_y \zeta_y - \Phi_z = 0,$$

$$[D] \quad \text{dynamic} \quad \Phi_t + \frac{1}{2} |\nabla \Phi|^2 + g \zeta = \frac{1}{2} U^2.$$

Both conditions are satisfied on the elevated free surface $z = \zeta$. Since the disturbance generated by the ship decays out at infinity, the fluid motion at infinity is identical with the uniform flow. If the ship is floating in ambient waves, the velocity potential at infinity is the sum of the uniform flow potential and the incident wave potential:

$$\Phi = U x + \Phi_w.$$

It is known that the solution of the boundary-value problem is not unique unless the radiation condition – generated waves propagate outwards only – is considered. The boundary conditions on the free surface are extremly non-linear, because, firstly, equations contain quadratic terms, and secondly, the equations are satisfied on the unknown surfcace which is determined after the problem is solved. The perturbation method is commonly used in solving such non-linear boundary-value problems. Slender-body theory is a typical application of perturbation analysis. The perturbation employed in slender-body theory is a singular perturbation, while thin-ship theory allows a regular perturbation. Slender-body theory assumes the perturbation expansion of the velocity potential around the $x$-axis which is taken as the longitudinal axis of the body. Since the velocity potential is singular in general on the $x$-axis, the expansion near the body
should be compatible with this singularity. Regular expansion, on the other hand, is applied to the flow field far from the body. The ratio of the transverse dimension of the body to the length is taken as the perturbation parameter which is assumed much smaller than unity. In the case of slender ships, the beam/length ratio is taken as the slenderness ratio. For the problem of oscillating ships or ships in ambient waves, the amplitude of oscillations is assumed small. Therefore we employ two small parameters, one of which is the beam/length ratio $\varepsilon$, and the other the ratio of wave amplitude to ship's length $\delta$. The wave length is assumed to be of the order of ship's length. Therefore the ship's length is usually taken as the reference length of the whole system.

Let us begin with the regular perturbation expansion which can be applied to the far field (far from the ship). The velocity potential is expanded by ascending power series with respect to the parameters $\varepsilon$ and $\delta$:

$$
\Phi = U x + (\varepsilon \phi^{(0)} + \varepsilon^2 \phi^{(02)} + \varepsilon^3 \phi^{(03)} + \ldots ) \\
+ \delta(\phi^{(10)} + \varepsilon \phi^{(11)} + \varepsilon^2 \phi^{(12)} + \ldots ) \\
+ \varepsilon^2(\phi^{(20)} + \varepsilon \phi^{(21)} + \varepsilon^2 \phi^{(22)} + \ldots ) + \ldots
$$

(7)

A similar expansion is applied to the free-surface elevation $\zeta$:

$$
\zeta = \varepsilon \zeta^{(01)} + \varepsilon^2 \zeta^{(02)} + \varepsilon^3 \zeta^{(03)} + \ldots \\
+ \delta(\zeta^{(10)} + \varepsilon \zeta^{(11)} + \varepsilon^2 \zeta^{(12)} + \ldots ) \\
+ \varepsilon^2(\zeta^{(20)} + \varepsilon \zeta^{(21)} + \varepsilon^2 \zeta^{(22)} + \ldots ) + \ldots
$$

(8)

Substitution of (7) in the Laplace equation (1), which holds for arbitrary $\varepsilon$ and $\delta$, leads to

$$
\nabla^2 \phi^{(ij)} = 0 \quad i, j = 0, 1, 2, 3, \ldots
$$

(9)

That means: each term of the expansion satisfies Laplace's equation. The boundary condition on the free surface is obtained by substituting (7) and (8) in (4) and (5). Since the free-surface conditions hold on the elevated surface $z = \zeta$, which is unknown, we perform the Taylor expansion about the horizontal plane $z = 0$, thus formulating the free-surface conditions on the undisturbed water surface:

$$
\begin{align*}
[K] & \quad (\varepsilon) \quad \zeta^{(01)}_t + U \zeta^{(01)}_x - \phi^{(01)}_x = 0 \\
& \quad (\delta) \quad \zeta^{(10)}_t + U \zeta^{(10)}_x - \phi^{(10)}_x = 0 \\
& \quad (\varepsilon \delta) \quad \zeta^{(11)}_t + U \zeta^{(11)}_x + \phi^{(01)}_x \zeta^{(01)}_x + \phi^{(10)}_x \zeta^{(10)}_x + \phi^{(10)}_y \zeta^{(10)}_y - \phi^{(01)}_y \zeta^{(01)}_y - \phi^{(11)}_x \zeta^{(11)}_x - \phi^{(11)}_y \zeta^{(11)}_y = 0 \\
& \quad (\varepsilon^2) \quad \zeta^{(02)}_t + U \zeta^{(02)}_x + \phi^{(01)}_x \zeta^{(01)}_x + \phi^{(01)}_y \zeta^{(01)}_y - \phi^{(02)}_x - \phi^{(02)}_y + \zeta^{(01)}_x = 0 \\
\ldots
\end{align*}
$$

(10)

$$
\begin{align*}
[D] & \quad (\varepsilon) \quad \phi^{(01)}_t + U \phi^{(01)}_x + g \zeta^{(01)} = 0 \\
& \quad (\delta) \quad \phi^{(10)}_t + U \phi^{(10)}_x + g \zeta^{(10)} = 0 \\
& \quad (\varepsilon \delta) \quad \phi^{(11)}_t + \phi^{(01)}_x \zeta^{(01)}_x + \phi^{(10)}_x \zeta^{(10)}_x + U \phi^{(01)}_x \zeta^{(01)}_x + U \phi^{(10)}_x \zeta^{(10)}_x + \phi^{(10)}_y \zeta^{(10)}_y + \phi^{(10)}_y \zeta^{(10)}_y + g \zeta^{(11)} = 0 \\
& \quad (\varepsilon^2) \quad \phi^{(02)} + \phi^{(01)}_x \zeta^{(01)}_x + U \phi^{(02)}_x + U \phi^{(01)}_x \zeta^{(01)}_x + \frac{1}{2} [(\phi^{(01)}_x)^2 + (\phi^{(01)}_y)^2 + (\phi^{(01)}_z)^2] \\
& \quad + g \zeta^{(02)} = 0 \\
\ldots
\end{align*}
$$

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However, as is shown by (15), $\phi^{(0)}$ is vanishing identically for slender ships. This simplifies the above equations to a great extent:

\[
\begin{align*}
[K] & \quad (\delta) \quad \zeta_x^{(10)} + U \zeta_x^{(10)} - \phi_x^{(10)} = 0 \\
 & \quad (\delta \epsilon) \quad \zeta_x^{(11)} + U \zeta_x^{(11)} - \phi_x^{(11)} = 0 \\
 & \quad (\epsilon^2) \quad U \zeta_x^{(02)} - \phi_x^{(02)} = 0 \\
[D] & \quad (\delta) \quad \phi_x^{(10)} + U \phi_x^{(10)} + g \zeta^{(10)} = 0 \\
 & \quad (\delta \epsilon) \quad \phi_x^{(11)} + U \phi_x^{(11)} + g \zeta^{(11)} = 0 \\
 & \quad (\epsilon^2) \quad U \phi_x^{(02)} + g \zeta^{(02)} = 0
\end{align*}
\]

(11)

Next, let us consider the expansion in the near field (near to the body). Since the conventional slender-body theory assumes that singularities are distributed along the longitudinal axis ($x$-axis), the Taylor expansion is not available around the $x$-axis. In order to develop the perturbation scheme, the technique of coordinate straining is introduced. Let us define the non-dimensional lengths $x^*, y^*, z^*$ by the relations

\[
x = \ell x^*, \quad y = by^*, \quad z = bz^*,
\]

(13)

where $\ell$ is half the length and $b$ the breadth of the body. It is assumed that the velocity field transformed into the $x^*, y^*, z^*$-space is isotropic, that means

\[
\frac{\partial \phi^{(ij)}}{\partial x^*} = O(1), \quad \frac{\partial \phi^{(ij)}}{\partial y^*} = O(1), \quad \frac{\partial \phi^{(ij)}}{\partial z^*} = O(1).
\]

Since $\ell$ is $O(1)$ and $b/\ell = \varepsilon$, the following relations are valid:

\[
\frac{\partial \phi^{(ij)}}{\partial x} = \frac{1}{\ell} \frac{\partial \phi^{(ij)}}{\partial x^*} = O(1), \quad \frac{\partial \phi^{(ij)}}{\partial y} = \frac{1}{\varepsilon} \frac{\partial \phi^{(ij)}}{\partial y^*} = O(\varepsilon^{-1}), \quad \frac{\partial \phi^{(ij)}}{\partial z} = \frac{1}{\varepsilon \ell} \frac{\partial \phi^{(ij)}}{\partial z^*} = O(\varepsilon^{-1}).
\]

Thus the differentiation with respect to $y$ or $z$ changes the order of magnitude by $\varepsilon^{-1}$. This fact is valid for the disturbance potential by the body, but it does not hold for the incident wave potential which is regular, so that (with $i = 1, 2, \ldots$)

\[
\frac{\partial \phi^{(io)}}{\partial x} = O(1), \quad \frac{\partial \phi^{(io)}}{\partial y} = O(1), \quad \frac{\partial \phi^{(io)}}{\partial z} = O(1).
\]

Since we regard $x = O(1), y = O(\varepsilon), z = O(\varepsilon)$ on the slender body, the following relations are valid:

\[
\frac{\partial x}{\partial n} = n_1 = O(\varepsilon), \quad \frac{\partial y}{\partial n} = n_2 = O(1), \quad \frac{\partial z}{\partial n} = n_3 = O(1).
\]

Now let us consider the boundary condition on the hull surface, when the ship makes oscillations with six degrees of freedom, which are composed of the translation $\dot{\xi}$ and the rotation $\vec{\alpha}$, both of $O(\delta)$. Then we have the following expansion of the hull condition:

\[
[H] \quad \ddot{\vec{n}} = (\dot{\zeta} + \vec{a} \times \vec{a}) \vec{n} + O(\delta^2)
\]

(14)

\[
O(\delta) = U n_1 + (\varepsilon \phi^{(01)}_n + \varepsilon^2 \phi^{(02)}_n + \ldots) + \delta (\phi^{(10)}_n + \varepsilon \phi^{(11)}_n + \varepsilon^2 \phi^{(12)}_n + \ldots)
\]

\[
O(\varepsilon) \quad O(1) \quad O(\varepsilon) \quad O(\delta) \quad O(\delta \varepsilon)
\]

Therefore
\[ \phi^{(01)} = 0 \]  
\[ \varepsilon \phi_n^{(02)} = -U n_1 \]  
\[ \delta (\phi_n^{(10)} + \varepsilon \phi_n^{(11)}) = [\hat{\xi} + \hat{\alpha} \times \hat{r} + U \hat{\alpha} + (\hat{\xi} + \hat{\alpha} \times \hat{r}) \cdot \nabla (\nabla \phi^{(02)})] \hat{n} \]  
\[ \delta (\phi_n^{(10)} + \varepsilon \phi_n^{(11)}) = [\hat{\xi} + \hat{\alpha} \times \hat{r} + U \hat{\alpha} + (\hat{\xi} + \hat{\alpha} \times \hat{r}) \cdot \nabla (\nabla \phi^{(02)})] \hat{n} \]

The last term in the square brackets on the right-hand side of (17) comes from the variation of \( \phi^{(02)} \) due to the shift of the hull surface. Next, the inner expansions of Laplace’s equation and of the boundary conditions on the free surface near the ship are obtained without difficulty:

\[ [L] \quad (\varepsilon^2) \quad \phi_{yy}^{(02)} + \phi_{xz}^{(02)} = 0 \]  
\[ (\delta \varepsilon) \quad \phi_{yy}^{(11)} + \phi_{xz}^{(11)} = 0 \]  
\[ [K] \quad (\varepsilon) \quad -\phi_{zz}^{(02)} = 0 \]  
\[ (\delta) \quad -\phi_{zz}^{(11)} - \phi_{zz}^{(02)} \xi^{(10)} = 0 \]  
\[ \ldots \quad \text{on} \quad z = 0 \]

\[ [D] \quad (\varepsilon^2) \quad U \phi_x^{(02)} + \frac{1}{2} [\phi_y^{(02)}]^2 + \phi_z^{(02)} = 0 \]  
\[ (\delta \varepsilon) \quad \phi_t^{(11)} + \frac{1}{2} \phi_y^{(10)} \phi_y^{(02)} + \phi_z^{(10)} \phi_z^{(02)} + \phi_y^{(11)} \phi_y^{(02)} + \phi_z^{(11)} \phi_z^{(02)} + g \xi^{(11)} = 0 \]  
\[ \ldots \quad \text{on} \quad z = 0 \]

The velocity potentials \( \phi^{(10)} \), \( \phi^{(11)} \) and \( \phi^{(02)} \) satisfy the linear boundary condition on the free surface which is obtained by eliminating \( \xi \) by means of combining [K] and [D] in (10) and (12):

\[ [F] \quad \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 \phi + g \frac{\partial \phi}{\partial z} = 0 \quad \text{on} \quad z = 0. \]

This is the well-known linearized free-surface condition for the free-surface flow around an oscillating ship with forward speed. Since \( \phi^{(10)} \) is the velocity potential due to incident waves, it is regarded as a known function. It satisfies [F] in the near field too, but the free-surface condition in the near field for the disturbance potentials \( \phi^{(02)} \) and \( \phi^{(11)} \) take other forms given by (20) through (23). It should be noted that the free-surface condition for \( \phi^{(11)} \) given by (21) and (23) is not homogeneous because of the term which comes from the product of contributions by \( \phi^{(02)} \) and \( \phi^{(10)} \), if forward speed is present. This may cause some complications in solving the boundary-value problem.

Here we define linearized potentials as follows:

Incident wave potential: \( \phi^{(W)} = \delta \phi^{(10)} = h \sqrt{g/K \exp[Kz - iK(x \cos \beta + y \sin \beta)] + i\omega t} \)

Steady potential: \( \phi^{(S)} = \varepsilon \phi^{(02)} \)

Periodical disturbance: \( \phi^{(R)} = \phi^{(D)} = \delta \varepsilon \phi^{(11)} \)

\( \phi^{(R)} \) is the radiation potential and \( \phi^{(D)} \) the diffraction potential. In (25), \( \omega \) is the circular frequency and \( K \) the wave number \( \omega^2/g \). The velocity potentials \( \phi^{(S)} \), \( \phi^{(R)} \) and \( \phi^{(D)} \) are harmonic functions in planes \( x = \text{constant} \) in the near field as shown by (18) and (19). The boundary condition on the hull surface is satisfied by these potentials at the mean position of the hull:

\[ \phi_n^{(S)} = -U n_1 \]

\[ \phi_n^{(R)} = [\hat{\xi} + \hat{\alpha} \times \hat{r} + U \hat{\alpha} + (\hat{\xi} + \hat{\alpha} \times \hat{r}) \cdot \nabla (\nabla \phi^{(02)})] \hat{n} \]

\[ \phi_n^{(D)} = -\phi_n^{(W)} \]
The boundary conditions on the free surface are satisfied on the plane \( z = 0 \). In the above argument, we have assumed that forward speed and frequency of oscillations are both \( O(1) \). Among the important consequences resulting from the formal expansion in the near field are the boundary conditions given by (20) and (21). That means the linearized potential in the near field for steady forward motion is identical with the potential when the free surface is a rigid plane wall, viz. the potential of a double body flow. If there is no forward speed, the periodical disturbance potential in the near field also satisfies the rigid-wall condition on the free surface. Therefore the solution becomes rather trivial. This fact will become an essential point of argument in this paper.

3. Longitudinal Oscillation Without Forward Speed (Radiation Problem)

If forward speed is zero, the equations become much simpler. Now, let us consider a slender ship making forced oscillations of small amplitude on the free surface under some external excitation in the \( \tau \) plane (longitudinal oscillations). The linearized potential has the form

\[
\phi^{(R)} = \phi e^{j\omega t}. \tag{31}
\]

The free-surface elevation can be written similarly:

\[
\zeta^{(R)} = \zeta e^{j\omega t} \tag{32}
\]

The Laplace equation and boundary conditions on the free surface in the near field are

\[
\begin{align*}
\text{[L]} & \quad \phi_{yy} + \phi_{zz} = 0 \\
\text{[K]} & \quad \phi_z = 0 \\
\text{[D]} & \quad i\omega\phi + g\zeta = 0
\end{align*} \tag{33} \tag{34} \tag{35}
\]

Corresponding equations in the far field are

\[
\begin{align*}
\text{[L]} & \quad \phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \\
\text{[K]} & \quad i\omega\zeta - \phi_z = 0 \\
\text{[D]} & \quad i\omega\phi + g\zeta = 0
\end{align*} \tag{36} \tag{37} \tag{38}
\]

By eliminating \( \zeta \) from (37) and (38), we obtain the linearized free-surface condition

\[
\text{[F]} \quad K\phi - \phi_z = 0 \tag{39}
\]

where \( K = \omega^2/g \). Since (34) is the rigid-wall condition and (33) the Laplace equation in two dimensions, the near-field potential can be expressed simply as

\[
\phi_N = \phi^{(2D)} + g(x), \tag{40}
\]

where \( \phi^{(2D)} \) is a plane harmonic function which satisfies the boundary condition on the hull surface and the rigid-wall condition on the plane \( z = 0 \):

\[
\frac{\partial \phi^{(2D)}}{\partial z} = 0. \tag{41}
\]

The solution for \( \phi^{(2D)} \) is quite elementary. The function \( g(x) \) is determined by matching with the far-field potential. The near-field potential \( \phi_N \) has an outer expansion at a considerable distance from the ship. Employing cylindrical coordinates

\[
y = R \sin \theta, \quad z = -R \cos \theta, \tag{42}
\]

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the outer expansion of \( \phi_N \) can be written as

\[
\phi_N = a_0 \ln R + \sum_{n=1}^{\infty} a_{2n} \frac{\cos 2n \theta}{R^{2n}} + \sum_{n=1}^{\infty} b_{2n} R^{2n} \cos 2n \theta + b_0
\]

(43)

because the motion is assumed symmetric in \( y \)-direction.

The potential in the far field is a three-dimensional harmonic function satisfying the linearized free-surface condition (39) in the plane \( z=0 \). It is expressed by means of Green's function associated with the linearized free-surface condition:

\[
G(x, y, z; x', y', z') = \frac{1}{r'} + \frac{1}{r} + 2K \int_0^\infty e^{\kappa z'} J_0(\kappa R') \frac{d\kappa}{\kappa - K - 2\pi iK} e^{K(z+z')} J_0(K R')
\]

(44)

where

\[
\begin{align*}
    r &= \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \\
    r' &= \sqrt{(x-x')^2 + (y-y')^2 + (z+z')^2} \\
    R' &= \sqrt{(x-x')^2 + (y-y')^2}.
\end{align*}
\]

\( J_0 \) is the Bessel function of the first kind, and the integral is meant to be the Cauchy principal value. The general form of the far-field potential is

\[
\phi_F = -\frac{1}{4\pi} \int_S \left[ \frac{\partial \phi}{\partial n} \bigg|_Q G(P, Q) - \phi \frac{\partial G(P, Q)}{\partial n_Q} \right] dS_Q
\]

(45)

where \( P \) means the field point and \( Q \) is a point on the surface \( S \). The asymptotic expansion in the far field gives the outer solution in the form

\[
\phi_F \simeq -\int_{-\ell}^{\ell} m(x') G(x-x', y, z; 0, 0, 0) \, dx' + O(\tau^{-2}),
\]

(46)

where \( m(x) \) is the source density given by

\[
m(x) = \frac{1}{4\pi} \int_C \frac{\partial \phi}{\partial n} \, ds,
\]

(47)

\( C \) being the girth of the hull section, and \( G(x, y, z; 0, 0, 0) \) is expressed as

\[
G(x, y, z; 0, 0, 0) = 2 \int_0^\infty e^{\kappa x} J_0(\kappa \bar{R}) \frac{\kappa d\kappa}{\kappa - K} = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-ikx} dk \int_{-\infty}^{\infty} \frac{\exp[z\sqrt{k^2 + m^2 - im\bar{y}}]}{\sqrt{k^2 + m^2 - K}} \, dm,
\]

(48)

where \( \bar{R} = \sqrt{x^2 + y^2 + z^2} \). The last term of (44) is included in the integral by the suitable choice of the integration contour around the pole. The inner expansion of the Fourier transform of the above function has been obtained by Ursell:

\[
G^*(k, y, z) = \int_{-\infty}^{\infty} e^{ikx} G(x, y, z; 0, 0, 0) \, dx
\]
\[
= 4 \int_0^\infty \frac{\cos m\gamma}{\sqrt{k^2 + m^2 - k}} e^{\frac{k}{\sqrt{k^2 + m^2}}} dm
\]

\[
= -4(1 + Kz)[\ln(\frac{1}{2}|k|R) + \gamma - Kz + Ky\theta]
\]

\[
+ [1 - (\frac{k}{K})^2]^{-\frac{1}{2}} \left\{ \begin{array}{ll}
\cos^{-1}(K/|k|) - \pi & \text{for } k > K \\
\cosh^{-1}(K/|k|) + i\pi & \text{for } k < K
\end{array} \right.
\]

(49)

Comparing the Fourier inversion of (49) with the outer expansion of the near-field solution (43), one can determine the undetermined coefficients in (43):

\[
a_0 = 4m(x)
\]

(50)

\[
b_0 = -2 \int_{-\ell}^{\ell} m'(x')\text{sgn}(x - x') \ln |x - x'| \, dx'
\]

\[
+ \pi K \int_{-\ell}^{\ell} m(x')[\mathcal{H}_0(K|x - x'|) + \mathcal{Y}_0(K|x - x'|) + 2i\mathcal{J}_0(K|x - x'|)] \, dx'.
\]

(51)

where \(\mathcal{Y}_0\) is the Bessel function of the second kind and \(\mathcal{H}_0\) is the Struve function. Since we have defined \(\phi^{(2D)}\) as the two-dimensional potential which satisfies the boundary condition on the hull surface and the rigid-wall condition on the plane \(z = 0\), the inner solution is given by

\[
\phi = \phi^{(2D)} - 2 \int_{-\ell}^{\ell} m'(x')\text{sgn}(x - x') \ln |x - x'| \, dx'
\]

\[
+ \pi K \int_{-\ell}^{\ell} m(x')[\mathcal{H}_0(K|x - x'|) + \mathcal{Y}_0(K|x - x'|) + 2i\mathcal{J}_0(K|x - x'|)] \, dx'.
\]

(52)

The two-dimensional potential is an even function of \(z\) and simply determined from the boundary condition on the hull surface, by known methods such as conformal mapping. The source density \(m(x)\) is determined from (47) which is also given by the hull boundary-condition. Therefore the solution is rather elementary.

If we consider heaving and pitching oscillations, we can write the vertical motion \(z\) and the angle of rotation \(\theta\) about the \(y\) axis:

\[
z = \ddot{z}e^{i\omega t}, \quad \theta = \ddot{\theta}e^{i\omega t}.
\]

(53)

Then the hull boundary-condition is written as

\[
\frac{\partial \phi^{(2D)}}{\partial \nu} = i\omega(\ddot{z} - \ddot{x}z) \cdot \nu_z
\]

(54)

where \(\nu\) is the normal to the section contour and \(\nu_z\) is the direction cosine with respect to the \(z\)-axis.

Joosen (1964) was the first to present numerical examples of this problem for heaving motion of a floating spheroid. Maruo and Tokura (1978) carried out computations for a Series 60 model, \(C_B = 0.7\). Fig. 3. shows added mass and damping coefficients for heave. The added mass turns to negative values and the damping coefficient increases without limits with increasing frequency. Curves resulting from strip theory are also shown. They provide an even better approximation of the actual values. Therefore the slender-ship formulation derived from the conventional slender-body theory is quite unsatisfactory for the purpose of prediction of hydrodynamic forces on oscillating ships except at very low frequencies.
4. Slender Ship in Uniform Forward Motion

The linearized velocity potential for uniform forward motion of a slender ship is given by $\phi^{(02)}$ in the expansion (7) because $\phi^{(01)} = 0$ by condition (15). Therefore one can write

$$\phi = \varepsilon^2 \phi^{(02)}.$$  \hspace{1cm} (55)

It satisfies the following equations in the near field:

$$[\mathcal{L}] \quad \phi_{yy} + \phi_{xz} = 0 \quad \text{on} \quad z = 0$$  \hspace{1cm} (56)

$$[\mathcal{K}] \quad \phi_z = 0 \quad \text{on} \quad z = 0$$  \hspace{1cm} (57)

$$[\mathcal{D}] \quad U \phi_z + \frac{1}{2} (\phi_y + \phi_x) + g' \zeta = 0 \quad \text{on} \quad z = 0$$  \hspace{1cm} (58)

$$[\mathcal{H}] \quad \phi_n = -Un_1 \quad \text{on} \quad S$$  \hspace{1cm} (59)

The solution in the near field (inner solution) takes the form

$$\phi = \phi^{(2D)} + g(x).$$  \hspace{1cm} (60)

Here $\phi^{(2D)}$ is a plane harmonic function determined by the boundary conditions

$$\frac{\partial \phi^{(2D)}}{\partial n} = -Un_1 \quad \text{on} \quad S,$$  \hspace{1cm} (61)

$$\frac{\partial \phi^{(2D)}}{\partial z} = 0 \quad \text{on} \quad z = 0.$$  \hspace{1cm} (62)

The outer expansion is similar to (43). The velocity potential in the far field is expressed using a Green’s function known as Kelvin source or Havelock source, which has the expression

$$G(x, y, z; x', y', z') = \frac{1}{r} - \frac{1}{r'}$$  \hspace{1cm} (63)

$$+ \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-ik(x-x')} dk \int_{-\infty}^{\infty} \frac{\exp[(z+z')\sqrt{k^2 + m^2} - im(y-y')]}{\sqrt{k^2 + m^2} - k^2/K_0} dm$$

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where $K_0 = g/U^2$. The integral with respect to $m$ is of a Cauchy kernel, and there is some indefiniteness. This is removed by choosing an integration contour which makes a detour around the singularities in such a way that the radiation condition is satisfied. The asymptotic expression for the velocity potential is similar to (46). The inner expansion of the Fourier transform $G^k(k,y,z)$ is obtained by replacing $K$ in Ursell’s expansion (49) by $k^2/K_0$. Then the inner expansion of the far-field potential is

\[
\phi \simeq 4m(x)\ln R - 2\int_{-\ell}^{\ell} m'(x')\text{sgn}(x - x')\ln(2|x - x'|)\,dx'
\]

\[
-\pi\int_{-\ell}^{\ell} m'(x')\left[\mathcal{H}_0(K_0|x - x'|) + \{2 + \text{sgn}(x - x')\}\mathcal{J}_0(K_0|x - x'|)\right]\,dx' + O(\varepsilon^3). \tag{64}
\]

The source density $m(x)$ is determined by the boundary conditions (61) and (62) resulting in the simple expression

\[
m(x) = \frac{U}{4\pi}S'(x) \tag{65}
\]

where $S(x)$ is the cross-sectional area of the ship. The wave resistance is given by

\[
R_W = -\frac{1}{2}\rho U^2\int_{-\ell}^{\ell} dx\int_{-\ell}^{\ell} S''(x)S''(x')\mathcal{J}_0(K_0|x - x'|)\,dx'. \tag{66}
\]

This expression is valid only when $S'(\pm\ell) = 0$, i.e. the ship has pointed ends. A correction for the end effect when the ship has vertical ends was considered by Maruo (1962). Fig. 4 is the curve of wave resistance of the parabolic hull. Curves by Michell’s thin ship theory and experimental results are also shown. The slender-ship formulation shows no improvement in comparison with thin-ship theory; it is even worse in the low speed range. Numerical results for a ship with pointed ends showed further aggravated discrepancies, Lewison (1963).
5. Linearization of the Radiation Problem by the Oscillation Amplitude

For an oscillating ship without forward speed, the linearized theory can be developed without assuming slenderness of the ship. The perturbation expansion is made by only one parameter $\delta$, the oscillation amplitude. The free-surface condition is linearized:

$$[F] \quad K \phi - \phi_z = 0 \quad \text{on} \quad z = 0. \quad (67)$$

The solution is expressed by using the wave source Green’s function (44) which was employed in the far-field expression for the slender ship in Section 3. The velocity potential is then expressed by Green’s theorem as

$$\phi_p = -\frac{1}{4\pi} \int_S \left[ \frac{\partial \phi}{\partial n}|_Q \left( G(P, Q) - \phi|_Q \frac{\partial G(P, Q)}{\partial n_Q} \right) \right] dS_Q. \quad (68)$$

Since $\frac{\partial \phi}{\partial n}|_Q$ is given by the boundary condition on the hull surface, (68) leads to an integral equation for $\phi$, and the solution is obtained by a numerical method. An alternative expression, which is particularly suitable to slender bodies, is the distribution of wave sources together with wave-free singularities along the longitudinal axis of the body. The expression for the wave source is given by (48):

$$G(x, y, z; 0, 0, 0) = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-ikx} dk \int_{-\infty}^{\infty} \frac{\exp\left[ z\sqrt{k^2 + m^2} - imy \right]}{\sqrt{k^2 + m^2 - K}} dm. \quad (69)$$

In order to apply the slender-body approximation, let us employ strained coordinates defined by (13),

$$x = \ell x^*, \quad y = by^*, \quad z = bz^*, \quad b/\ell = \varepsilon,$$

and put

$$k = k^*/\ell, \quad m = m^*/b.$$

Then the Fourier transform of the wave source becomes

$$G^*(k^*, y^*, z^*) = 4 \int_0^\infty \frac{\cos m^* y^*}{\sqrt{\varepsilon^2 k^* + m^* - Kb}} e^{i \varepsilon k^* z^* + m^*} dm^*. \quad (70)$$

Since we are seeking the solution which is valid at higher as well as at lower frequencies, the order of magnitude of $Kb$ is not specified. The expansion of (70) with respect to $\varepsilon$ leads to

$$G^*(k^*, y^*, z^*) = 4 \int_0^\infty \frac{\cos m^* y^*}{m^* - Kb} e^{m^* z^*} dm^* + O(\varepsilon^2)$$

$$= 4 \int_0^\infty \frac{\cos m y}{m - K} e^{m z} dm + O(\varepsilon^2). \quad (71)$$

The first term on the right-hand side is identical with the wave source in two dimensions. It is well known that the integral is indefinite because of the pole of the integrand; it is made
definite by the radiation condition that only the outgoing phase is present at infinite distance. However, the above expression is valid only in the near field, so that the radiation condition valid at infinity is not applicable. Keeping this indefiniteness, the inner solution has the expansion

$$
\phi = a_0 \int_0^\infty \frac{\cos my}{m-K} e^{mz} \, dm
$$

$$
- \sum_{n=1}^{\infty} a_{2n} \left[ \frac{\partial^{2n-1}}{\partial z^{2n-1}} \left( \frac{z}{z^2 + y^2} \right) + K \frac{\partial^{2n-2}}{\partial z^{2n-2}} \left( \frac{z}{z^2 + y^2} \right) \right]
$$

$$
+ b_0 (1 + Kz) + b_2 [z^2 - y^2 + Kz(z^2 + y^2)] + \ldots
$$

$$
\simeq a_0 [\ln KR + KR \cos \theta (1 - \ln KR - \gamma) + KR \theta \sin \theta]
$$

$$
+ \sum_{n=1}^{\infty} a_n \left[ \frac{\cos 2n\theta}{R^{2n}} + \frac{K}{2n-1} \frac{\cos(2n-1)\theta}{R^{2n-1}} \right]
$$

$$
+ b_0 (1 - KR \cos \theta) + O(R^2 \ln R). \quad (72)
$$

The wave-free singularities are given by the summation term, and $b_0, b_2, \ldots$ are indeterminate coefficients corresponding to the indeterminateness of the integral in $(71)$. When $Kb$ is small, the inner solution is expressed by $(52)$. It is expressed in the form

$$
\phi = a_0(x) [\ln KR + KR \cos \theta (1 - \ln KR) + KR \theta \sin \theta]
$$

$$
- \frac{1}{2} (1 - KR \cos \theta) \int_{-\epsilon}^{\epsilon} a_0'(x') \text{sgn}(x - x') \ln(2K |x - x'|) \, dx'
$$

$$
- \frac{\pi}{4} (1 - KR \cos \theta) \int_{-\epsilon}^{\epsilon} a_0'(x') [H_0(K|x - x'|) + \mathcal{H}_0(K|x - x'|)] + 2iJ_0(K|x - x'|)] \, dx'
$$

$$
- \sum_{n=1}^{\infty} a_{2n} \left[ \frac{\cos 2n\theta}{R^{2n}} + \frac{K}{2n-1} \frac{\cos(2n-1)\theta}{R^{2n-1}} \right] + O(R^2 \ln R). \quad (73)
$$

It is found that $(72)$ and $(73)$ are equivalent up to $O(\epsilon)$ but for the regular terms which are indefinite in $(72)$. Since these terms must match in both expressions, we set

$$
b_0 = -a_0(x)(\ln 2K + \gamma) - \frac{1}{2} \int_{-\epsilon}^{\epsilon} a_0'(x') \text{sgn}(x - x') \ln |x - x'| \, dx'
$$

$$
+ \frac{\pi}{4} K \int_{-\epsilon}^{\epsilon} a_0'(x') [H_0(K|x - x'|) + \mathcal{H}_0(K|x - x'|)] + 2iJ_0(K|x - x'|)] \, dx'. \quad (74)
$$

As is shown in the previous section, the expansion on $(72)$ or $(73)$ is valid only when $Kb$ is small. At large wave numbers such as $Kb = O(1)$, terms representing the three-dimensional effect decay out, and the velocity potential is given by the solution for two-dimensional motion at each transverse plane. Then let us define $\phi^{(2D)}$ for the solution of the two-dimensional problem such as

$$
\phi^{(2D)} = -a_0 \int_0^\infty \frac{\cos my}{m-K} e^{mz} \, dm - \pi i e^{Kz} \cos Ky
$$

$$
- \sum_{n=1}^{\infty} \frac{a_{2n}}{(2n-1)!} \left[ \frac{\partial^{2n-1}}{\partial z^{2n-1}} \left( \frac{z}{z^2 + y^2} \right) + K \frac{\partial^{2n-2}}{\partial z^{2n-2}} \left( \frac{z}{z^2 + y^2} \right) \right]. \quad (75)
$$
This satisfies the radiation condition at infinity. If we express the velocity potential for three-dimensional motion in the form

\[
\phi = \phi^{(2D)}_0 - a_0(1 + Kz)(\gamma + \pi i) - \frac{1}{2} a_0(1 + Kz) \int_{-\ell}^\ell a'_0(x') \text{sgn}(x - x') \ln(2K|x - x'|) \, dx'
\]

\[
+ \frac{\pi}{4} K(1 + Kz) \int_{-\ell}^\ell a_0(x')[\xi_0(K|x - x'|) + \nu_0(K|x - x'|) + 2i \mathcal{J}_0(K|x - x'|)] \, dx',
\]

it is readily shown that the expansion of the above formula coincides with (73) up to \(O(\varepsilon)\) at small wave-numbers \(Kb = O(\varepsilon)\), while at high wave-numbers terms representing the three-dimensional effect decay out, and the velocity potential tends asymptotically to the two-dimensional solution \(\phi^{(2D)}_0\). Therefore (76) is valid both at small and large wave-numbers. Thus the formula is regarded as the interpolation between two extreme cases and may be called the interpolation theory.

\[
\frac{a}{\rho L^3} \quad \text{ADDED MASS}
\]

\[
\frac{b}{\rho g L^{5.3}} \quad \text{DAMPING}
\]

\[
\text{----- PRESENT METHOD}
\]

\[
\text{---- ORIGINAL SLENDER}
\]

\[
\text{SHIP THEORY}
\]

\[
\text{----- STRIP THEORY}
\]

\[
\bullet \quad \text{EXPERIMENT}
\]

\[\text{Fig. 5. Added mass and damping coefficient in heave}\]

The 'unified theory' proposed by Newman is based on a slightly different idea. The Fourier transform of the inner Green's function defined in (71) is indeterminate because the radiation condition at infinity in the two-dimensional problem is not applicable to the inner solution of the slender ship. The indeterminateness is represented by the standing-wave potential which can be superimposed arbitrarily. Therefore, the complete solution is expressed as

\[
\phi = -a_0 \left[ \int_0^\infty \frac{\cos my}{m - K} e^{mz} \, dm + A \pi i e^{Kz} \cos K y \right]
\]

\[
- \sum_{n=1}^\infty \frac{a_{2n}}{(2n - 1)!} \left[ \frac{\partial^{2n-1} z}{\partial z^{2n-1}} \frac{z}{z^2 + y^2} + K \frac{\partial^{2n-2} z}{\partial z^{2n-2}} \frac{z}{z^2 + y^2} \right].
\]

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The indeterminate coefficient is determined by matching with the inner expansion of the far-field solution (73). The result is

\[
A = -a_0 \gamma - \frac{1}{2} \int a_0(x') \text{sgn}(x - x') \ln(2K|x - x'|) \, dx' \]

\[+ \frac{\pi}{4} K \int_a^{\ell} a_0(x') [H_0(K|x - x'|) + J_0(K|x - x'|)] \, dx'.
\] (78)

If \( e^{Kz} \) is replaced by \( 1 + Kz \) in (77), the result becomes identical to (76). Therefore the difference between the two theories is not substantial, but the three-dimensional effect represented by the unified theory decays out more rapidly in high frequencies than the interpolation theory.

Fig. 5 shows added mass and damping coefficients of the Series 60 model, \( C_B = 0.7 \), predicted by the interpolation theory, Maruo and Tokura (1978). Results of computations by strip theory and by the formulation given in Section 3, as well as measurements by Gerritsma and Beukelman (1964) are also shown. Plausible agreement between results of the present theory and the measured data is observed.

6. Extending Unified Theory to Finite Forward Speed

As discussed in the previous section, perturbation analysis is comparatively easy, if forward speed is absent. When forward speed is introduced, the perturbation scheme changes because the condition of small disturbance is not warranted by the small amplitude of oscillation alone. The slenderness of the body plays an important role in the perturbation scheme as in the case of steady forward motion. Formal perturbation expansion according to the conventional slender-body technique results in a consistent formulation for the linearized solution. However, difficulties appear if finite speed is introduced, because numerical examples have revealed that no useful results are provided by this kind of formulation. Apart from the rigorous application of perturbation analysis, the idea of extending the unified theory to oscillation with forward speed has been attempted by Newman and his school. The theory begins with the linearized free-surface condition (24) in the far field:

\[
(\omega + U \frac{\partial}{\partial x})^2 \phi + g \phi_x = 0 \quad \text{on } z = 0.
\] (79)

Green's function associated with the above boundary condition is

\[
G(x, y, z; x', y', z') = \frac{1}{r} - \frac{1}{r'}
\]

\[+ \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-ik(x-x')} \frac{dk}{\sqrt{k^2 + m^2}} \int_{-\infty}^{\infty} \exp\left[\sqrt{k^2 + m^2} + \ln\left(\frac{\sqrt{k^2 + m^2} - (\omega - Uk)^2/g}{\sqrt{k^2 + m^2} - (\omega - Uk)^2/g}\right) \right] \, dm.
\] (80)

The Fourier transform of \( G(x, y, z; x', y', z') \) is

\[
G^*(k, y, z) = 2 \int_{-\infty}^{\infty} \frac{\exp\left[\sqrt{k^2 + m^2} \, - \, i \gamma \right]}{\sqrt{k^2 + m^2} - (\omega - Uk)^2/g} \, dm.
\] (81)

The inner expansion of the above is obtained by replacing \( K \) in Ursell's expansion (49) by \((\omega - Uk)^2/g = K^*\):

\[
G^*(k, y, z) \sim -4(1 + K^*z) \ln \left(\frac{1}{2} K^* R\right) + \gamma - K^*z + K^*y \theta
\]

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\[ + \left( 1 - \frac{k^2}{K^2} \right)^{- \frac{1}{2}} \begin{cases} \cos^{-1}(K^*/|k|) - \pi \\ \cosh^{-1}(K^*/|k|) - \pi \text{sgn}(\omega - U K) \end{cases} \]

\[ = -4(1 + K^*z)[\ln K^*R + \gamma + \pi i - K^*z + K^*y\theta - f^*(k)]. \quad (82) \]

Assume an extension of the far-field Green's function to the near field, and employ strained coordinates for the slender body as before. Then the Fourier transformed Green's function becomes

\[ G^*(k^*, y^*, z^*) = 2 \int_{-\infty}^{\infty} \frac{\exp[\sqrt{\varepsilon^2 k^{*2} + m^2} z - i m y]}{\sqrt{\varepsilon^2 k^{*2} + m^2} - \sqrt{\varepsilon^2 b^2/g - \varepsilon^2 k^*/g^2}} \, dm^*. \quad (83) \]

Here \( K^* = \omega^2 b/g \) is not necessarily regarded as a small quantity of \( O(\varepsilon) \). The expansion with respect to \( \varepsilon \) leads to the near-field approximation for \( G^*(k^*, y^*, z^*) = G(k, y, z) \):

\[ G^*(k, y, z) \approx 4 \int_{0}^{\infty} \frac{\cos m y}{m - K} \, dm + A \varepsilon K^z \cos Ky + O(\varepsilon^{\frac{3}{2}}). \quad (84) \]

The coefficient \( A \) in the second term represents the indeterminateness of the inner solution which is determined by matching with the far-field solution. The procedure employed by the unified theory for zero forward speed gives

\[ A = \frac{4}{\pi i} f^*(k) \quad (85) \]

with

\[ f^*(k) = \ln \left( \frac{2K^*}{|k|} \right) + \frac{\pi i}{\sqrt{|1 - k^2/K^*|}} \begin{cases} \cos^{-1}(K^*/|k|) - \pi \\ \cosh^{-1}(K^*/|k|) - i \pi \text{sgn}(\omega - U k) \end{cases} \quad \text{if } k > K, \quad \text{if } k < K. \quad (86) \]


However, several points seem to cause problems:

1. The formulation is not derived by rational perturbation.
2. The error is \( O(\varepsilon^{\frac{3}{2}}) \) if \( U = O(1) \), \( \omega^2 b/g = O(1) \); that is not small.
3. The flow pattern in each transverse section is similar to that at zero forward speed, except for the standing wave which is still two-dimensional.
4. At the limit of zero frequency, the fluid motion tends to the original slender ship formulation for steady forward motion discussed in Section 4, which is unsatisfactory.

This situation may be improved slightly by assuming (Maruo and Matsunaga 1983) the Froude number to be \( O(\varepsilon^{\frac{3}{2}}) \) and taking terms up to \( O(\varepsilon) \). By this assumption, the effect of forward speed appears as a correction term to the zero forward speed case; however, predicted results deviate from measured data at higher Froude numbers.

7. Solution in the Time Domain

There is another approach to solving the problem for oscillating slender ships at finite forward speed, which is somewhat intuitive. In the theories discussed so far, the coordinate
system is moving with the ship. If one takes the coordinate system fixed in space, and if the ship is moving in still water, the fluid motion is unsteady even in the uniform forward motion. Then the analysis becomes an initial value problem in the time domain. This method was employed originally for transient motions of a floating body in two dimensions. Seemingly, Ogilvie (1974) was the first to propose application of this method to slender ships with forward speed. Chapman (1975) applied this technique to a flat plate in lateral oscillations at forward speed. Saito and Takagi (1978), Adachi and Ohmatsu (1980) and Yeung and Kim (1981) attempted to extend this method to longitudinal oscillations of slender ships.

Fig. 6. Coordinate system for the solution in time domain

Consider a slender ship moving in negative x-direction with forward speed \( U \). The relation between the coordinate system \( X,Y,Z \) which is fixed in space, and the system \( x,y,z \) moving with the ship as used in the previous analysis, is \( X = x - Ut \), \( Y = y \), \( Z = z \). The position of the ship changes in time with respect to the new coordinate system. At a certain instant, the coordinate plane \( YZ \) intersects the hull at a section \( S \) as shown in Fig. 6. Since the ship is slender, the fluid motion in the \( YZ \)-plane is assumed to be two-dimensional. Therefore, the governing equation is the Laplace equation in two dimensions:

\[
[L] \quad \phi_{YY} + \phi_{ZZ} = 0. \tag{87}
\]

The boundary condition on the free surface with respect to the coordinates \( X,Y,Z \) becomes

\[
[F] \quad \phi_{tt} + g \phi_Z = 0 \quad \text{on} \quad Z = 0. \tag{88}
\]

The hull surface is expressed with respect to \( x,y,z \) by

\[
y = F(x,z,t). \tag{89}
\]

Then the boundary condition on the hull surface in the new coordinates is

\[
[H] \quad \phi_n = -Un_1 + [-U \alpha_2 + \dot{\alpha}_3 + \ddot{\alpha}_2(X + Ut)]n_3 \quad \text{on} \quad Y = F(X + Ut, Z, t). \tag{90}
\]

The time-dependent Green's function in the \( YZ \)-plane for the boundary condition (88) is given by

\[
G(Y, Z; Y', Z', t - \tau) = \delta(t - \tau) \ln(R/R')
\]
\[
-2gH(t-\tau) \int_0^\infty e^{ik(Z+Z')} \cos k(Y-Y') \sin \sqrt{gk}(t-\tau) \frac{dk}{\sqrt{gk}} \tag{91}
\]

where \(\delta(t-\tau)\) is Dirac's delta function and \(H(t-\tau)\) is Heaviside's unit step function. If we express the fluid motion by a source distribution on the hull surface, the inner potential in the plane \(X = 0\) is given by

\[
\phi(Y, Z, t) = \int_{-\infty}^{t} d\tau \int_{C(\tau)} \sigma(Y', Z', \tau)G(Y, Z; Y', Z', t-\tau) \, ds \tag{92}
\]

where \(ds\) is taken along the contour of the hull section which changes with time. It is noted that the result is reduced to strip theory if forward speed is not present. Since the solution is purely two-dimensional, it does not include the genuine three-dimensionality of the fluid motion.

8. Application of the Interpolation Concept to the Problem of Steady Forward Motion (Maruo 1982)

The Green's function in the far field for steady forward motion is the Kelvin source. The original slender-ship formulation assumes a distribution of Kelvin sources along the longitudinal \((x)\) axis of the ship, and the near-field solution is obtained from the asymptotic expansion around this axis as presented by Tuck. Now let us examine the asymptotic behaviour of the Kelvin source potential along its track. The Kelvin source potential is

\[
\phi = \frac{K_0}{\pi} \int_{-\pi}^{\pi} d\theta \int_0^\infty \exp[i\alpha - ikx \cos \theta - iky \sin \theta] \frac{d\kappa}{\kappa^2 - K_0^2} \\
-2K_0 \int_{-\pi}^{\pi} e^{K_0 \sec^2 \theta \sin(K_0x \sec \theta) \cos(K_0y \sec \theta \tan \theta) \sec^2 \theta} \, d\theta. \tag{93}
\]

If normalization by the characteristic wave number \(K_0 = g/U^2\) is introduced, put

\[
K_0x = X, \quad K_0y = Y, \quad K_0z = Z, \quad \kappa \cos \theta = K_0 k, \quad \kappa \sin \theta = K_0 m, \tag{94}
\]

and we obtain

\[
\phi = \frac{K_0}{\pi} \int_{-\infty}^{\infty} e^{-ikx} dk \int_{-\infty}^{\infty} \exp[\sqrt{k^2 + m^2}Z - imY] \frac{dm}{k^2 - \sqrt{k^2 + m^2}} \\
-4K_0 \int_1^\infty e^{2u^2} \sin(Xu) \cos(Yu \sqrt{u^2 - 1}) \frac{u}{\sqrt{u^2 - 1}} \, du. \tag{95}
\]

We substituted \(K_0 \sec \theta = u\) in the last term of (93). If \(X = O(1), Y = O(\varepsilon)\) and \(Z = O(\varepsilon)\), the asymptotic expression is obtained by putting \(Y = \varepsilon Y', Z = \varepsilon Z'\) and taking the limit \(\varepsilon \to 0\). Then at sufficient distance from the singularity, the expression is approximated by

\[
\phi \simeq \frac{K_0}{\pi} \int_{-\infty}^{\infty} e^{-ikx} dk \int_{-\infty}^{\infty} \frac{dm}{k^2 + \sqrt{k^2 + m^2}} - 4K_0 \int_1^\infty \sin(Xu) \frac{u}{\sqrt{u^2 - 1}} \, du \\
= -\frac{2K_0}{X} + \pi K_0 [\mathcal{H}_1(X) - \mathcal{Y}_1(X) - \frac{2}{\pi}] + 4\pi K_0 \mathcal{Y}_1(X) \quad \text{for } X > 0, \tag{96}
\]

\[
= \frac{2K_0}{X} - \pi K_0 [\mathcal{H}_1(X) - \mathcal{Y}_1(X) + \frac{2}{\pi}] \quad \text{for } X < 0.
\]

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The asymptotic expression at \( X = O(\varepsilon^{\frac{1}{2}}) \) near the singularity is obtained by putting \( X = \varepsilon^{\frac{1}{2}} x^* \), \( k = \varepsilon^{-\frac{1}{2}} k^* \), \( m = \varepsilon^{-1} m^* \), \( u = \varepsilon^{-\frac{1}{2}} v^* \), and taking the limit \( \varepsilon \to 0 \). Then the asymptotic expression in the original coordinates \( X, Y, Z \) is given by

\[
\phi \sim \frac{K_0}{\pi} \int_{-\infty}^{\infty} e^{-ikX} \, dk \int_{-\infty}^{\infty} \frac{\exp[i|m|z-imY]}{k^2-|m|} \, dm - 4K_0 \int_{0}^{\infty} e^{2u^2} \sin(Xu) \cos(Yu^2) \, du
\]

\[
= -8K_0 \int_{-\infty}^{\infty} e^{2u^2} \sin(Xu) \cos(Yu^2) \, du \quad \text{for} \; X > 0,
\]

\[
= 0 \quad \text{for} \; X < 0.
\]

(97)

Thus the Kelvin source has different forms of asymptotic expressions near and far from the singularity. The expression for \( X > 0 \) of (97) becomes \(-8K_0/X\) on the \( x \)-axis, which is compatible with the asymptote of (96) at small \( X \). Then we have an expression which is uniformly valid at both large and small values of \( X \):

\[
\phi \sim -8\sqrt{K_0} \int_{-\infty}^{\infty} e^{2z} \cos(v^2y) \sin(v\sqrt{K_0x}) \, dv
\]

\[
+ \pi K_0[\mathcal{H}_1(K_0x) + 3\mathcal{Y}_1(K_0x)] + \frac{6}{\pi} - 2K_0 \quad \text{for} \; X > 0,
\]

\[
- \pi K_0[\mathcal{H}_1(K_0x) - \mathcal{Y}_1(K_0x)] + \frac{6}{\pi} - 2K_0 \quad \text{for} \; X < 0.
\]

(98)

The integral in (98) can be expressed by the complex Fresnel integral:

\[
\int_{-\infty}^{\infty} e^{\nu^2} \cos(v^2y) \sin(v\sqrt{K_0x}) \, dv = -\text{Im}e^{iK_0\nu^2/4Z} \sqrt{\frac{\pi}{2Z}} F\left(x\sqrt{\frac{K_0}{2\pi Z}}\right)
\]

where \( Z = y - iz \) and

\[
F(x) = C(x) + iS(x) = \int_{0}^{\infty} e^{ivu^2/2} \, dv.
\]

(99)

(98) is compared with the kernel function of (64). The singularity of the latter on the \( x \)-axis is logarithmic, which means a source, while (98) has no source singularity but an essential singularity on the \( x \)-axis. This is due to the fact that the expansion which is used in the original slender-ship theory does not hold near the singularity. An implication of this fact is that it is not possible to employ the line distribution of Kelvin sources along the longitudinal axis as a representation of the slender ship. Therefore we have to assume a source distribution over the hull surface for the steady forward motion of a slender ship. It is obvious that the term of lowest order in the perturbation expansion of the boundary condition on the free surface is linear with respect to the disturbance velocity potential. Then let us begin with the linearized free-surface condition for steady forward motion,

\[
\frac{\partial^2 \phi}{\partial x^2} + K_0 \frac{\partial \phi}{\partial x} = 0 \quad \text{on} \; z = 0.
\]

(100)

Green's function for this boundary condition is given by (63) as

\[
G(x, y, z; x', y', z') = \frac{1}{r} - \frac{1}{r'} \quad (101)
\]

\[
+ \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-ik(x-z')} \, dk \int_{-\infty}^{\infty} \frac{\exp[(z + z')\sqrt{k^2 + m^2} - im(y - y')]}{\sqrt{k^2 + m^2} - k^2/K_0} \, dm.
\]

The term corresponding to the last term on the right-hand side of (95) is omitted for simplicity, so that the integral with respect to \( m \) should be carried out along a suitable contour in order to satisfy the radiation condition. The Fourier transform of the Green's function is
\[ G^*(k, y, z) = 2K_0(kR) - 2K_0(kR') + \int_{-\infty}^{\infty} \frac{\exp[\pm\sqrt{k^2 + m^2 - im\tilde{y}}]}{\sqrt{k^2 + m^2 - k^2/K_0}} \, dm \]

where \( R = \sqrt{(y - y')^2 + (z - z')^2} \quad R' = \sqrt{(y - y')^2 + (z + z')^2} \)
\[ \tilde{y} = y - y' \quad \tilde{z} = z + z' \quad K_0 = \text{Macdonald function} \]

In order to find a suitable expression in the near field, let us employ strained coordinates as before, \( x = \ell x^*, y = by^*, z = bz^*, \) and assume \( x^*, y^*, z^* = O(1), \) \( b/\ell = \varepsilon \ll 1. \) We also put
\[ k = k^*/\ell, \quad m = m^*/b, \quad \tilde{y} = b\tilde{y}^*, \quad \tilde{z} = b\tilde{z}^*, \quad R = bR^*, \quad R' = bR'^*. \]

Then the Fourier-transformed Green's function can be written as
\[ G^*(k^*, y^*, z^*) = 2K_0(\varepsilon k^*R^*) - 2K_0(\varepsilon k^*R'^*) + 2 \int_{-\infty}^{\infty} \frac{\exp[\pm\sqrt{\varepsilon k^2 \cdot m^2 + im\tilde{y}^*}]}{\sqrt{\varepsilon k^2 + m^2 - \varepsilon k^2/K_0\ell}} \, dm^*. \]

The asymptotic expansion of the integral in (103) depends on the magnitude of \( \varepsilon k^2/K_0\ell = b\kappa^2/K_0, \) as shown in the case of oscillation at zero forward speed. If the wave number \( k \) is sufficiently small such as \( k^* = O(1), \) we put \( m^* = \varepsilon m^{**} \) and obtain
\[ G^*(k^*, y^*, z^*) \approx -2\ln(R^*/R'^*) + 2 \int_{-\infty}^{\infty} \frac{\exp[\pm\sqrt{\varepsilon k^2 \cdot m^2 + im\tilde{y}^*}]}{\sqrt{\varepsilon k^2 + m^{**^2} - k^2/K_0\ell}} \, dm^{**}. \]

The expansion of the integral is of Ursell's type as given in Section 4. Therefore its Fourier inversion results in the velocity potential (64) which is the original formulation of the slender ship. However, it is not valid near the singularity where large wave numbers are dominant as shown in the expansion of the Kelvin source. At large wave numbers of the order \( \varepsilon^{-\frac{1}{2}} \) we put \( k^* = \varepsilon^{-\frac{1}{2}} k^{**} \) and let \( \varepsilon \to 0: \)
\[ G^*(k^*, y^*, z^*) \approx -2\ln(R^*/R'^*) + 4 \int_{0}^{\infty} e^{m^{**}z^*} \frac{\cos m^{**}\tilde{y}^*}{m^{**^2} - k^{**2}/K_0\ell} \, dm^{**} + O(\varepsilon k^{**2}) \]

or, in the original coordinate system,
\[ G^*(k, y, z) \approx -2\ln(R/R') + 4 \int_{0}^{\infty} e^{mz} \frac{\cos my}{m - k^2/K_0} \, dm + O(\varepsilon k^2). \]

Thus we have two expressions which are equivalent to (96) and (97) for the Kelvin source but for the logarithmic term, which represents the source and its negative image. Following the interpolation concept, the Fourier transformed Green's function is expressed by the uniformly valid approximation
\[ G^*(k, y, z) \approx -2\ln(R/R') + 4 \int_{0}^{\infty} e^{mz} \frac{\cos my}{m - k^2/K_0} \, dm + 4\pi i \text{sgn}(k) e^{\pm\kappa y^2/K_0} \cos(\tilde{y}k^2/K_0) \]
\[ + (1 + \tilde{z}k^2/K_0)g^*(k) \]

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where $g^*(k)$ is the Fourier transform of $g(z)$ given by
\[
g(z) = -K_0[H_1(K_0x) + 3Y_1(K_0x)] + 6/z - 2K_0 \quad \text{for } x > 0, \quad (108)
\]
\[
K_0[H_1(K_0x) - Y_1(K_0x)] + 2/x - 2K_0 \quad \text{for } x < 0.
\]
The term $1 + \frac{z}{K_0}$ in the last term of (107) corresponds to $(1 + Kx)$ in (76) of the interpolation theory for the oscillation of zero forward speed.

The fluid motion around a ship hull is expressed by distributing sources over the hull surface. The line distribution along the waterline is omitted as of higher order because of the slender-ship condition. Then one can write
\[
\phi = -\int \sigma G(x, y, z; x', y', z') \, dS. \quad (109)
\]
Applying the Fourier convolution with (107) gives the near-field potential for the slender ship:
\[
\phi = 2 \int_{C(x)} \sigma \ln(R/R') \, ds
\]
\[
-8\sqrt{K_0} \int_{-t}^{t} dz' \int_{C(x')} \sigma \, ds \int_{0}^{\infty} e^{m(z + z')} \cos m(y - y') \sin \sqrt{K_0} m(x - x') \, \frac{dm}{\sqrt{m}}
\]
\[
- \int_{-t}^{t} g(x - x') \, dz' \int_{C(x')} \sigma \, ds + \frac{z}{K_0} \frac{d^2}{dz^2} \int_{-t}^{t} g(x - x') \, dz' \int_{C(x')} \sigma \, ds \quad (110)
\]
where $C(x)$ is the contour of the ship section at $x$, and $ds$ is taken along the contour $C(x)$. The last term may be omitted as of higher order, but it contributes to the numerical result when $K_0z$ is not very small. For applying the boundary condition of the hull surface, it is convenient to decompose the potential into several parts:
\[
\phi = \phi_1 + \phi_2 + F(x) - (z/K_0) F''(x) \quad (111)
\]
where
\[
\phi_1 = 2 \int_{C(x)} \sigma \ln(R/R') \, ds \quad (112)
\]
\[
\phi_2 = -8\sqrt{K_0} \int_{-t}^{t} dz' \int_{C(x')} \sigma \, ds \int_{0}^{\infty} e^{m(z + z')} \cos m(y - y') \sin \sqrt{K_0} m(x - x') \, \frac{dm}{\sqrt{m}} \quad (113)
\]
\[
F(x) = - \int_{-t}^{t} g(x - x') \, dz' \int_{C(x')} \sigma \, ds \quad (114)
\]
Then the boundary condition on the hull surface is written as
\[
\frac{\partial \phi}{\partial n} = -U n_1 = \frac{\partial \phi_1}{\partial n} + \frac{\partial \phi_2}{\partial n} - \frac{F''(x)}{K_0} n_3. \quad (115)
\]
The last term on the right-hand side can be omitted because of higher order. Then this equation leads to the integral equation to determine the source density $\sigma$:
\[
2\pi \sigma + 2 \int_{C(x)} \sigma \frac{\partial}{\partial n} \ln(R/R') \, ds = -U n_1 - \frac{\partial \phi_2}{\partial n} \quad (116)
\]
Since $\frac{\partial \phi_2}{\partial n}$ is determined by the source density at sections upstream, the boundary-value problem is reduced to the parabolic type, and the solution is much easier than in the original problem of elliptic type.

![Graph showing wave resistance coefficient](image)

**Fig. 7.** Computed and measured wave resistance coefficient

![Graph showing wave profile](image)

**Fig. 8.** Wave profile alongside the hull

Numerical examples of wave pattern, wave resistance and pressure distribution on the hull surface calculated by the present theory have been provided by Song et al. (1988). Fig. 7 shows the wave-resistance coefficient vs. Froude number, Fig. 8 the wave profile along the side of a model of a Series 60, $C_B = 0.6$ hull form. In these computations, an approximate treatment has been employed for the evaluation of the last term in (94). Results of measurements in the towing tank as well as computations by Michell's thin-ship theory are also shown in these figures. Plausible agreement between measurement and computation by the present theory is observed.
9. Extending Interpolation Theory to an Oscillating Ship at Finite Forward Speed

The argument for the steady forward motion can be extended to oscillating ships at finite forward speed. Green's function for this case is given by (80). Its Fourier transform is

\[
G^*(k, y, z) = 2K_0(kR) - 2K_0(kR') + 2 \int_{-\infty}^{\infty} \frac{\exp[\sqrt{k^2 + m^2} - iny]}{\sqrt{k^2 + m^2} - (\omega - Uk)^2 / g} \, dm. \tag{117}
\]

Using strained coordinates, we have

\[
G^*(k^*, y^*, z^*) = 2K_0(\varepsilon k^* R^*) - 2K_0(k^* R^*) + 4 \int_{0}^{\infty} \frac{\exp[z^* \sqrt{\varepsilon^2 k^* - m^*} + m^* \tilde{y}^*]}{\sqrt{\varepsilon^2 k^* - m^*} - (\omega - U^2 / g + \sqrt{\varepsilon k^* K_0})^2} \, dm^*. \tag{118}
\]

When \( k^* = O(\varepsilon^{1/2}) \), corresponding to the region near the singularity \( x - x' = O(\varepsilon^{1/2}) \), the Fourier-transformed Green's function has an asymptotic expression which is obtained by \( \varepsilon \to 0 \):

\[
G^*(k, y, z) \approx -2\ln(R/R') + 4 \int_{0}^{\infty} \frac{\cos m\tilde{y}}{m - (\omega - U - k)^2 / K_0} \, dm \\
+ 4\pi \text{sgn}(k - \omega / U) e^{i(k - \omega / U)^2 / K_0} \cos[\tilde{y}(k - \omega / U)^2 / K_0] \tag{120}
\]

When \( k^* = O(1) \), corresponding to the region far from the singularity, the expression for \( G^* \) becomes

\[
G^*(k, y, z) \approx \frac{2}{\pi} f^*(k)[1 + z(k - \omega / U)^2 / K_0] \tag{122}
\]

where \( f^*(k) \) is given by (86). Then the asymptotic expression which is uniformly valid near and far from the singularity can be written as

\[
G^*(k, y, z) \approx -2\ln(R/R') + 4 \int_{0}^{\infty} \frac{\cos m\tilde{y}}{m - (\omega - U - k)^2 / K_0} \, dm \\
+ 4\pi \text{sgn}(k - \omega / U) e^{i(k - \omega / U)^2 / K_0} \cos[\tilde{y}(k - \omega / U)^2 / K_0] \\
+ g^*_1(k)[1 + z(k - \omega / U)^2 / K_0] \tag{123}
\]

where \( g^*_1(k) \) is determined from \( f^*(k) \) by subtracting the values of the second and third terms on the right-hand side of (123) on the \( x \)-axis. Thus the near-field potential for the oscillating slender ship is obtained in a way similar to the case of steady forward motion:

\[
\phi = 2 \int_{\partial(z)} \sigma \ln(R/R') \, ds \\
-2\sqrt{K_0} \int_{-t}^{t} dx' \int_{\partial(z')} \sigma ds' \int_{0}^{\infty} e^{m(x + x') - \omega(x - x') / U} \cos m(y - y') \sin \sqrt{K_0 m(x - x')} \, dm / \sqrt{m} \\
- \int_{-t}^{t} g_1(x - x') \, dx' \int_{\partial(z')} \sigma ds - \frac{2}{g} (\omega - U \frac{\partial}{\partial x})^2 \int_{-t}^{t} g_1(x - x') \, dx' \int_{\partial(z')} \sigma ds \tag{124}
\]

where \( g_1(x) \) is the Fourier inversion of \( g_1^*(k) \). This expression coincides with the time-domain solution by changing variables as \( x = -\omega + Ut, x' = -\omega + Ut \) and omitting the third and fourth term on the right-hand side. It is reduced to the result of the interpolation theory (76)
when $U = 0$. Yeung and Kim (1984) have derived a formula which has much resemblance with the above as an extension of the solution in the time domain. A slight difference is observed, since the former is not based on perturbation analysis.

10. Conclusions

The development of slender-body theory applied to ship hydrodynamics is reviewed. The idea of applying slender-body theory to ship problems is by no means new. In contrast to the rapid progress and remarkable achievements of slender-body theory in aerodynamics, progress of the theory of slender ships has been rather sluggish. This seems to be due to the fact that slender-ship formulation in the earlier stage was not able to provide results which were compatible with measured data. The difficulty involved in the slender-body formulation for free-surface flows around a ship hull seems to be attributed to the singular behaviour of the wave-source potential which is much more complicated than ordinary singularities which appear in the unbounded flow. Since slender-body theory depends on the rational method of singular perturbation, the nature of the singularity is of primary importance. Only in recent years this fact has been recognized. There have been several attempts which depend mainly on rather intuitive approaches. However, only a rational perturbation analysis will provide a consistent result.

Summarizing the development of the theory of slender ships, the following conclusions can be derived:

1. The slender-ship formulation should be based on rational perturbation analysis.

2. The lowest order solution of formal expansion by the slenderness ratio, assuming ordinary singularity, yields unrealistic results in the prediction of hydrodynamic forces for periodical as well as steady motion.

3. The formulation which takes account of the higher wave-number contribution of the kernel function provides much improvement. Compared to strip theory, it shows better agreement with measured data, especially at low frequencies, in the prediction of hydrodynamic forces and moments of an oscillating ship at zero forward speed.

4. The concept employed for oscillation at zero forward speed such as the interpolation theory may be extended to the case of finite forward speed with and without oscillations.

5. The slender-ship formulation for steady forward motion seems to be able to provide better prediction of wave resistance than thin-ship theory.

6. The theory of oscillating slender ships with finite forward speed will not be completed until it provides numerical results compatible with measured data.

7. A rational formulation which has practical feasibility for slender ships with forward speed, oscillating in ambient waves, has not yet been developed.

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