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Seventh Georg Weinblum Memorial Lecture

SURFACE WAVES FROM THE RAY POINT OF VIEW

by

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The Seventh Georg Weinblum Memorial Lecture:

SURFACE WAVES FROM THE RAY POINT OF VIEW

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INTRODUCTION

In commenting on a paper of Bessho (1966), Georg Weinblum noted:

"Science is subject to fashion as much as other human activities. Recently the thin ship and surrogates have completely dominated the field, but in the twenties (and earlier) the pressure system has been considered as being an equally important hydrodynamic model (at least in principle) as the Michell ship, especially suitable for picturing fast shallow-draft and planing vessels. By Dr. Bessho's paper a sound equilibrium has been established. The present speaker had emphasized the similarity of the Hogner and the Michell integral (Zamm, 1930) and thus inspired Sir Thomas Havelock to derive the simple relation between source-sink distributions \( a \) and pressure systems \( p \)

\[ 4\pi \sigma \rho a = c \frac{2p}{\partial x} \]  

These days the fashion in ship waves has been very much with so-called low-speed theories, which can be implemented through digital computation. The question has arisen, Keller (1979), as to the nature of true asymptotic low speed theory. In that paper he proposed a ray theory. In the present paper, which I have prepared especially for this lecture, I have chosen to explore the ray theory and to begin by combining it with a very old fashioned subject, and one which early attracted the attention of Weinblum himself, the waves made by a moving pressure patch. In this case, assuming light loading, linearizing assumptions are valid and the theory takes a simple form. It is therefore very useful for sharpening our tools and insight.

After that start we tackle the ship problem, as Keller already has. We made no assumptions concerning the thickness of the ship. We repeat some of his findings. We also find some waves issuing from a limited portion aft and from the ends. We have formulae for these waves. At the ends the situation is, however, ambiguous because of insufficient knowledge of the displacement flow giving rise to the waves.

A few words concerning ray theory. Its antecedents are found in geometrical optics. In dispersive systems it arises with group velocity as a product of asymptotic integration and is inherent in Kelvin (W. Thomson) (1887) and Havelock (1908), and later works, and then much later in Stoker (1957) who considered the wave pattern created by a ship moving in a curved trajectory. All of these assumed no displacement flow in the water. But in problems of optics and acoustics, the inhomogeneity of the medium had long been considered. For dispersive systems, at least for ship waves, this was first discussed and the basic relations given by Ursell (1960), and independently by Whitham (1961). At about the same time, the fundamentals of the interaction between waves and currents were laid out in a series of important papers: Longuett-Higgins and Stewart (1960, 1961) and Whitham (1960, 1962); see also the discussion in Phillips (1966). The basic assumption of the ray theory is that the waves are short in comparison to the scale over which the flow changes (it is this assumption which is at question near the ends of the ship), so that the waves may be assumed to have locally the same dispersive relation as in undisturbed water (it is this assumption which Eggers (1981) questions at the bow of a ship).

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A detailed study of the implications of ray theory for ship waves was begun and later extended by Keller (1974, 1979), while the actual application was begun by Inui and Kajitani (1977) who assumed the waves generated as in linear theory and utilized ray theory to calculate the bending of the rays. Yim (1981) has later made extensive ray tracing studies in the same spirit.

Here we are motivated to understand how ships generate waves, in this case in the true low speed limit; as many questions are eventually raised as are answered.

1. WAVES GENERATED BY A STEADY MOVING PRESSURE PATCH IN THREE DIMENSIONS

Introduction

The problem of the waves generated by a moving real ship contains serious non-linear features. On the other hand, the waves generated by a pressure patch of finite size moving on top of the water surface can perhaps be treated using linearizing assumptions, provided that the magnitude of the pressures are suitably small. And the solution to this problem can yield valuable understanding of wavemaking, just as Kelvin's treatment in 1887 of a concentrated pressure patch. This is certainly the reason why Georg Weinblum (1930) undertook to study this problem. Much later it became of some importance in connection with the performance of air cushion vehicles like the Hovercraft, and was studied by several workers, notably Nick Newman (1962).

Here we study this problem using ray (asymptotic) techniques and obtain some important results; then we go on to the case of a real ship, which is considerably more complicated; but non-linear effects can be treated with interesting results. These ray techniques which we use, first in the linear pressure patch case and then in the non-linear ship case, are generally applicable in the limit of small Froude number.

The usual linearizing assumptions which we employ in the pressure patch problem are: i) the deep water waves propagate locally as progressive waves of small amplitude, for which the dispersion relation between the wave frequency, \( \omega \), and wave number, \( k \), is: \( \omega = gk \); ii) waves are generated at each point on the free surface under the pressure patch, and the amplitude of the generated waves is proportional to the excess pressure, \( p \), associated with the patch; iii) the waves generated at each point add linearly; iv) the waves, once generated, propagate over the water surface as if it were at rest. The last assumption is decisive and is equivalent to assuming that the moving pressure patch does not induce any significant motion in the water, aside from the waves themselves. This is certainly not true in the case of displacement ships, where the water is forced to go around the ship as well as to issue waves; we shall correct for this vital difference later.

We imagine a patch of constant pressure moving over the water at a speed \( U(t) \), from right to left along the x-axis, Figure 1.1. Our technique will be to observe the waves, \( \eta(t) \), arriving at an observer point, \( P(x,z) = t \), at a distance of many wavelengths from the moving pressure patch. We assume to begin with that these waves can arrive along any ray passing from the observer point to the pressure patch (later we show that only distinct rays contribute at any time, \( t \)). We designate the rays by their angle of inclination, \( \beta \), to the horizontal axis. In our present approximation, since no motion occurs in the water to bend the rays, they are straight lines.

The Far Field Plane Wave Spectrum

An elemental radial wave is generated as if by a concentrated imposition of pressure \( p \) (a delta function) at each point of the water over which the pressure patch passes. Those waves generated along a fixed ray bundle (origin at \( \xi \)) of mean ray angle \( \beta \) and width, \( (\xi - \xi')d\beta \), and observed at the fixed point, \( \xi(x,z) \), can be represented as an integration of the generator points, \( \xi' \), along the mean ray, and over time, \( t' < t \):

\[
\frac{dn}{dp} (\xi, t, \beta) = R \int dt' \int \frac{p}{p} [A(\xi - \xi')] e^{i(k(\xi - \xi') - \omega(t - t'))} d\xi' \;
\quad \xi_g(k) = \frac{t - t'}{t' - t} \tag{1.1}
\]

where \( t \) are the intersections of the ray with the upper and lower boundaries of the patch, and \( C = \omega d\xi \). The function \( [A] \) is found from the asymptotic (ray) theory solution for the wave due to a concentrated imposition of pressure. It is, see Havelock (1908) or Lamb (1932):

\[
[A] = g(t - t')^3/2^7/2\pi(\xi - \xi')^4 = g/2^7/2\pi(\xi - \xi')C_g^3 \tag{1.2}
\]
The result of this collection of waves generated within the ray bundle is a plane wave of wave number $k$ and amplitude, $\text{Amp}(\beta)$:
\[
\frac{dn}{d\beta} = R \cdot [{\text{Amp}(\beta)}]e^{i[k\zeta - wt]}; \quad \text{far field} \tag{1.3}
\]

It was originally pointed out by Thomas Havelock (1934), that the far field wave may be represented by a distribution (spectrum, $dn/d\beta$) of such waves:
\[
\eta(\zeta,t) = \frac{\pi}{\int_0^\pi} \frac{dn}{d\beta} (\zeta,t,\beta) \, d\beta \tag{1.4}
\]

and the wave resistance (in the case of uniform motion) given simply as:
\[
\text{Res.} = \pi\rho U^2 \int_{\pi/2}^{\pi} \frac{[R \cdot \{Amp^2(\beta)\} + I \cdot \{Amp^2(\beta)\} \cos^3 \beta] \, d\beta}{n/2} \tag{1.5}
\]

a result we shall refer to later. In consequence, the problem of determining wave resistance is equivalent to the determination of $dn/d\beta$, [1.1], in the far field ($\zeta \to \infty$).

The Boundary Sources

The integral, [1.1], has no stationary phase points so its asymptotic form can be determined through repeated integration by parts. The result can be represented as a sum of terms with coefficients $k, k^2, \text{etc.}$ In the present case, where the observer is many wavelengths from the generator point, the first term in this series is dominant and:
\[
\frac{dn}{d\beta} (\zeta,t,\beta) = R \cdot \int_0^\infty \frac{p_0}{\rho k} \{[A(\zeta - \xi_0', t-t')][(\xi-t')^*]e^{ikA(\zeta - \xi_0'; t-t')]}(\xi-t)^* \cdot \frac{f}{k} \, dt' \tag{1.6}
\]

where $f = k(\xi-t^*) - \omega(t-t')$, and $\xi^*$ refer to the intersection of the ray with the patch boundary at time $t'$. We see, [1.6], that waves seem to originate only on the boundary of the pressure patch.

Radiation from the Boundary: Ray Theory

The integration of these wave generators (i.e. in $t'$) represented by [1.6] is facilitated by the application of Kelvin's method of stationary phase, see Stoker (1957); this method assumes asymptotic conditions; i.e. that the wave length is much shorter than the range of integration and that $A$ varies slowly enough. We recall for reference, if:
\[
b(\zeta) = R \cdot \int_{\tau}^{\tau_2} a(\zeta,t')e^{i\psi(\zeta,t')} \, dt'
\]

then,
\[
b(\zeta) \equiv R \cdot \sum_p^\infty \frac{i\psi(\zeta,t_p) + \pi/4}{\text{sgn} \psi_p} \cdot a(\zeta,t_p) \left[ \frac{2\pi}{\psi_p} \right]^{1/2} + \\
\left. + R \cdot \sum_p^\infty \frac{i\psi(\zeta,t_p)}{a(\zeta,t_p) \left[ \frac{\Gamma(1/3)}{\psi_p} \frac{6}{[\psi_p]} \right]^{1/3} \right]
\]

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where \( \psi_1 = \frac{\partial u}{\partial t} (t_0) = 0 \), defines the points \( t' = t_0 \), where the dominant contribution to the integral arises when \( u'' = 0 \), and where \( \psi_0 = \frac{\partial u}{\partial t} = 0 \) defines the points \( t = t_0 \), where the dominant contribution arises when \( u'' = 0 \); i.e. \( \frac{\partial u}{\partial t} \) sum only over \( s \), and when \( \frac{\partial u}{\partial t} = 0 \) sum only over \( p \). In this latter case, rays focus by bunching and the resultant wave is more observable. This condition is called a caustic.

Upon applying Kelvin's method to the integration of the boundary waves, [1.6], we find the main contribution arising from points where (stationary phase point):

\[
\frac{df_{u,t}}{dt} = \frac{d}{dt} \left( k(t-t') - w(t-t') \right) = 0
\]

or,

\[
\frac{dr^*}{dr} = \frac{w}{k} = \frac{C_p(k)}{k}
\]

[1.8]

where \( C_p \) is the phase velocity of the wave; this condition defines the wave number of the wave arising at each point of the path boundary; i.e. the phase velocity of the outgoing boundary wave is equal to the velocity of the boundary along the ray.

We have so far considered an arbitrary velocity of the pressure patch. When the patch moves with constant horizontal speed, \( U_0 \), then all of the waves in the resulting pattern must be stationary in body coordinates (the frame of reference moving with the patch) and it follows that the phase speed, \( C_p \), of the wave traveling along a ray at angle \( \beta \) (Figure 1), is:

\[
C_p(k) = -U_0 \cos \beta
\]

[1.9]

This follows from the fact that the frequency of wave encounter to an observer traveling with velocity \( V_{obs} \) is:

\[
k \cdot \frac{V_{obs} \cdot e^{-C_p t}}{}
\]

Notice that for waves moving outward toward the observer point above the patch, \( C_p > 0 \), so that \( \pi/2 < \beta < \pi \). These waves may be classified in the usual way: those traveling on rays closest to the vertical (divergent waves) are short, while those on rays closest to the horizontal (transverse waves) are longest.

This condition of stationarity, [1.9], when combined with the stationary phase condition, [1.8], leads to a relationship between the local ray angle, \( \beta(k) \), and the local patch angle measured from the horizontal, \( \alpha^* \). First it may be shown taking into account the cutting angle, \( \alpha \), that:

\[
\frac{dr^*}{dt} = \frac{U_0 \tan \alpha^*}{\cos \left( \tan \beta - \tan \alpha^* \right)} = \frac{U_0 \sin \alpha^*}{\sin(\beta - \alpha^*)}
\]

[1.10]

and then combining [1.8-1.10] we finally find that the wave number vector is normal to the patch boundary:

\[
(\beta - \alpha^*) = \pi/2
\]

[1.11]

This is a result we might have expected. It is well known that in the case of non-dispersive wave systems (optics or acoustics), that the signal is primarily due to excitation from the point on the body which lies closest to the observer and therefore arrives on the ray normal to the body.

We note: each and every point of a smooth patch boundary produces (at a fixed observer point above the patch) a single wave (single \( k \), corresponding to \( \beta \)); however each wave (\( k \)) receives a contribution from all points sharing the same patch boundary angle, \( \alpha^* \); the transverse waves arise particularly from the blunt ends and the divergent waves from the moderately sloped sides. The waves traveling to an observer above the patch will originate from the upper forward and rear aft sectors of the patch boundary, see Figure 1.2. For smooth shapes, then, each wave will be excited at two generator points, one in each of the contributing sectors, provided that a normal to the ray exists (for boundaries with non-blunt ends, the range of \( \beta \) will be limited). Finally a simple relation exists for the phase velocity at any point: \( C_p = U_0 \sin \alpha^* \).