Institut für Schiffbau der Universität Hamburg

MANEUVERING HYDRODYNAMICS OF ELLIPSOIDAL FORMS

by

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Abstract

The hydrodynamical forces and moments acting on a tri-axial ellipsoid moving in an incompressible and inviscid fluid are analysed. The rigid ellipsoid is allowed to move in the most general manner with time dependent velocity and six degrees of freedom. The force and moment expressions are obtained by applying the Lagally theorem to the image singularities system representing the body in the presence of exterior disturbances. First expressions for the Lagally force and moment acting on a maneuvering ellipsoid in an unbounded medium are derived and then these expressions are generalized to include the effect of an exterior source moving in an arbitrary manner. It is also shown how the Lagally expressions for an exterior source can be used to obtain closed form expressions for the hydrodynamical forces and moments acting on a maneuvering ellipsoid in the presence of arbitrary exterior disturbance. The analysis which is based on the application of ellipsoidal harmonics is demonstrated in a simple case of propeller-hull interaction. Here the motion of the ellipsoid is restricted to the major axis and the propeller at the stern is represented by an isolated sink in accordance with Dickmann's model. Practical expressions for the thrust deduction coefficient, wake fraction and propeller induced vibration are then derived for ellipsoidal, spheroidal and spherical hulls.

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1. INTRODUCTION

We are now celebrating the 25th anniversary of the Institut für Schiffbau. This is an occasion to remember Professor Weinblum, the renowned first director of the Institute. Among the many imprints Professor Weinblum left in the field of ship theory were his studies of Maneuvering Hydrodynamics of general ship forms and of ellipsoidal forms in particular. Professor Weinblum advocated the use of ellipsoidal forms as an approximation for general ship forms and his often repeated statement "the ellipsoid is God's gift to naval architects!" (quoted by Newman (1972)) has stimulated many valuable studies. His approach to ship maneuvering extensively used the Lagally theorem of which Professor Weinblum was one of the main promotors and one of the first to realize its power and versatility. For these reasons the present paper which analyses general maneuvering of ellipsoids by employing the Lagally theorem is dedicated to Professor Weinblum's memory in this jubilee year of the Institut für Schiffbau.

Due to the ever increasing ship size and traffic density there is currently a growing interest in the area of ship maneuvering especially in confined and shallow water. In spite of this the contributions from theoretical hydrodynamics to the field of ship maneuvering have been limited as pointed out by the
Maneuverability Committee of the 14th International Towing Tank Conference held in 1975. Following the recommendation of this conference, it was decided to analyse the problem of a general ellipsoid maneuvering in the proximity of a general disturbance (wall, free-surface, body, propeller, etc.). As is customary in maneuvering studies, the effect of compressibility, viscosity and Froude number will be neglected and the free surface will be replaced by the double-model approximation.

The determination of the hydrodynamical forces and moments acting on a maneuvering ship in a bounded or disturbed fluid medium is indeed a hard task. The problem may be simplified by using a tri-axial ellipsoid as an approximation to the ship hull taking advantage of some analytic tools which are particularly tailored for ellipsoided shapes. The idea of replacing a ship hull by an "equivalent ellipsoid" was first suggested by Weinblum (1936) and later used by Eggers (1960), Wu and Chwang (1974), and Miloh and Landweber (1977).

The present paper considers the most general time-dependent motion of a tri-axial ellipsoid with six degrees of freedom in a disturbed fluid medium. The fluid is assumed to be inviscid and incompressible and the effect of fluid separation is ignored. The mathematical formulation employs the classical theory of ellipsoid harmonics (Hobson 1955) together with some recent contributions to potential flow theory (Miloh 1973, 1974). These theories are particularly appealing in view of the recent progress in computerizing the numerical computation of the various Lamé functions (Walter 1971).

The concept of "force and moment influence functions" is introduced to calculate the forces and moments experienced by a maneuvering ellipsoid in the proximity of an exterior disturbance which may be represented by an arbitrary distribution of singularities. First general expressions for the forces and moments acting on a maneuvering ellipsoid in the presence of an exterior
moving source with a time dependent output are derived. It is then shown how these "force and moment influence functions" may be used to determine the hydrodynamical forces and moments acting on an ellipsoid moving in the proximity of a general prescribed system of singularities representing, wall, free-surface, solid bodies, etc. This concept is somewhat analogous to the use of the Green's function of a unit source to determine the velocity potential of an arbitrary distribution of singularities by formulating an integral equation.

The general expression for the forces and moments acting on an ellipsoid in the proximity of an exterior source are obtained by applying the Lagally theorem to the image singularity system as described in Section 3 for the case of a stationary ellipsoid in the presence of exterior source, and in Section 4 for maneuvering ellipsoid in unbounded medium. The direct application of the Lagally theorem to determine the hydrodynamical forces and moments acting on a maneuvering ellipsoid in unbounded medium is demonstrated in Sections 5 and 6, respectively. Finally, expressions for the forces and moments experienced by an ellipsoid moving with six degrees of freedom in the proximity of an exterior unsteady source with arbitrary spacial motion are derived in Sections 7 and 8. It is also shown how these "influence functions" may be used to obtain closed-form expressions for the hydrodynamical forces and moments acting on the ellipsoid in the proximity of a general prescribed singularity system representing fluid or solid boundaries.

In Section 9 the general expressions thus derived are demonstrated to calculate the force experienced by the ellipsoid in a relatively simple case, namely an allipsoid translating in the direction of its major axis in the presence of an exterior sink with time dependent output lying on the same axis. The results of this example may be found useful in analyzing propeller-hull interaction, in accordance with Dickmann's isolated-sink model of the propeller. Other parameters such as potential wake fraction, thrust deduction coefficient and propeller induced vibratory force, are also briefly discussed.
2. MATHEMATICAL BACKGROUND

In order to solve the Laplace equation in a domain bounded (internally or externally) by a tri-axial ellipsoidal surface, it is convenient to formulate the problem in ellipsoidal coordinates. The ellipsoidal (confocal) coordinates are defined by the solutions of the following cubic equation in \( \chi \);

\[
\frac{x_1^2}{\chi^2} + \frac{x_2^2}{\chi^2 - h^2} + \frac{x_3^2}{\chi^2 - k^2} = 1
\]

for a fixed value of the Cartesian coordinates \((x_1, x_2, x_3)\). Both \(h\) and \(k\) are related to the three semi-axes of the ellipsoid i.e. \(a_1, a_2, \) and \(a_3\) in the \(x_1, x_2\) and \(x_3\) directions respectively by

\[
k^2 = a_1^2 - a_3^2; \quad h^2 = a_1^2 - a_2^2
\]

such that \(a_1 > a_2 > a_3\)

The three roots of (1), here denoted by \(\rho, \mu\) and \(\nu\), are so chosen that

\[
\infty > \rho^2 > k^2; \quad k^2 > \mu^2 > h^2; \quad h^2 > \nu^2 > 0
\]

The three surfaces \(\rho = \text{constant (ellipsoids)}, \mu = \text{constant (hyperboloids of one sheet)}\) and \(\nu = \text{constant (hyperboloids of two sheets)}\) form a spacial triply orthogonal coordinate system. In such a coordinate system the canonical ellipsoid with the semi-axes \((a_1, a_2, a_3)\) is simply given by \(\rho = a_1\).

The transformation between the ellipsoidal and the Cartesian coordinates is given in Hobson (1955, p. 454)
\[ x_1^2 = \frac{\rho^2}{h^2} \frac{\mu^2}{k^2} v^2 \quad (4) \]

\[ x_2^2 = \frac{\left( \rho^2 - h^2 \right) \left( \mu^2 - h^2 \right) \left( v^2 - h^2 \right)}{h^2 \left( k^2 - h^2 \right)} \quad (5) \]

\[ x_3^2 = \frac{\left( \rho^2 - k^2 \right) \left( k^2 - \mu^2 \right) \left( k^2 - v^2 \right)}{k^2 \left( k^2 - h^2 \right)} \quad (6) \]

The three linearizing factors of the above transformation in the \( \rho, \mu \) and \( v \) directions are here denoted by \( \bar{h}_\rho, \bar{h}_\mu, \) and \( \bar{h}_v \) respectively and are given by

\[ \bar{h}_\rho^2 = \frac{\left( \rho^2 - \mu^2 \right) \left( \rho^2 - v^2 \right)}{\left( \rho^2 - h^2 \right) \left( \rho^2 - k^2 \right)} \quad (7) \]

\[ \bar{h}_\mu^2 = \frac{\left( \rho^2 - \mu^2 \right) \left( \mu^2 - v^2 \right)}{\left( \mu^2 - h^2 \right) \left( k^2 - \mu^2 \right)} \quad (8) \]

\[ \bar{h}_v^2 = \frac{\left( \rho^2 - v^2 \right) \left( \mu^2 - v^2 \right)}{\left( h^2 - v^2 \right) \left( k^2 - v^2 \right)} \quad (9) \]

A normal (separable) solution of the Laplace equation at the point \( M(\rho, \mu, v) \) which is regular at the origin may be written as

\[ P_n^m (M) = P_n^m (\rho, \mu, v) = E_n^m (\rho) \quad E_n^m (\mu) \quad E_n^m (v) \quad (10) \]

where \( E_n^m \) denotes the Lamé function of the first kind of order \( n \) and degree \( m \). Here both \( n \) and \( m \) are positive integers satisfying \( 2n + 1 \geq m \geq 1 \).