Weak-interaction theory of ocean waves

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1. Introduction

Ocean wave research covers a broad range of topics including the theoretical analysis of the basic processes of wave growth and decay, semi-empirical methods of wave-forecasting and engineering problems related to the effects of waves on ships, structures and beaches. We shall be concerned here primarily with the state of the sea as such, rather than the secondary effects of waves.

The increased interest in ocean waves in the past two decades was originally stimulated by the wave prediction problem. Since dynamical wave theory was virtually nonexistent, wave forecasting evolved for many years as an essentially empirical art. However, the latest developments show promise of a stronger interaction with dynamical wave theory, which has made considerable advances in recent years. A general theoretical framework has emerged, enabling a rational discussion of both the prediction problem and the dynamical processes determining the local energy balance of the waves. An assessment of the present state of the field may be useful before proceeding to the more detailed measurements and computations which will be needed to place the theoretical framework on a sounder quantitative basis.

The first prediction methods by Svedrup and Munk (1943, unclassified 1947) and Suthons (1945) were based on a simplified parametric description of both wind and wave fields. Empirical relationships were established between the characteristic parameters of each field. The introduction of a wave spectrum in the prediction methods of Neumann (1953) and Pierson, Neumann and James (1955) represented an important conceptual advance. However, the reliability of these methods and the alternative prediction formulae suggested by Darbyshire (1955, 1959), Bretschneider (1959), Roll and Fischer (1956), Burling (1959) and others was
still basically limited by the parametric description of the wind fields, which was retained in the new methods, but was now no longer matched to the more sophisticated description of the wave field.

The present forecasting methods use a complete description of both the wind and wave fields and are based on the numerical integration of the radiative transfer equation. The approach was pioneered by Gelci and collaborators (cf. Gelci et al., 1956, Gelci and Cazalé, 1962, Fons, 1966) and has been developed independently by Baer (1962), Pierson et al., (1966) and Barnett (1966). The source functions used in these methods are still largely empirical. However, a closer interaction with dynamical wave theory may be expected in the future. The functional form of most terms in the source function can now be predicted theoretically, although extensive measurements and computations are still needed to fill in many details.

Dynamical wave theory is the statistical theory of the local interactions of the wave field with the atmosphere and ocean. The first significant contributions to this problem were Phillips' (1957) and Miles' (1957) theories of wave generation, which yielded rigorous transfer expressions for certain aspects of the wave-atmosphere interactions which had been discussed previously by Eckart (1953), Jeffreys (1925, 1926), Wuest (1949), Lock (1954) and others. A further contribution was the determination and computation of the energy transfer due to non-linear wave-wave interactions (Phillips, 1960, Hasselmann, 1960, 1962, 1963 a,b).

We shall see in the following that these processes may be regarded as particular applications of a general theory of weak interactions, which yields the energy transfer for all expansible interactions between the wave field and the atmosphere and ocean.
The lowest-order set of transfer expressions for wave-atmosphere interactions consists of the Phillips and Miles processes, a non-linear correction to Miles' process, and wave-turbulence interactions. Present data suggests that the wave-turbulence interactions may be the most important of the four.

The interactions between waves and the ocean are formally very similar to wave-atmosphere interactions. The lowest-order processes consist of parametric damping by mean currents, scattering by large and medium-scale turbulence and parametric damping by small-scale turbulence. The last process may be interpreted as an eddy viscosity. A further application of the general interaction theory is the diffusion due to waves, but this will not be considered here.

The major part of this paper will be devoted to the development and application of the weak interaction theory. The theory yields the source functions for the radiative transfer problem, which will be discussed briefly in the first section. The interaction and transfer problems are complementary aspects of the complete problem of determining the state of the sea for a given wind field. Although we shall consider only ocean waves in detail, the basic concepts are applicable to all random wave fields. We shall accordingly present the theory first for an arbitrary set of interacting fields, following Hasselmann (1966, 1967a). Since the emphasis is on developing a general approach, we cannot do adequate justice to many specific contributions to the subject; we refer in this respect to the more extensive expositions of Kinsman (1965) and Phillips (1966).
2. The radiation balance

2.1 Representation of the wave field

Ocean waves are a statistical phenomenon; it is meaningful to consider only average properties of the wave field. In practice, the mean values are determined either as time or spatial averages. For theoretical purposes, it is more convenient to consider the mean values as averages over a hypothetical ensemble of fields. Our averages will be defined in this latter sense. The two definitions are equivalent if the field is either statistically stationary or homogeneous, i.e., if all mean quantities are invariant under either time or (horizontal) spatial translations.

To a first approximation, an ocean wave field is both stationary and homogeneous. This implies that the dynamical processes changing the state of the field are weak, and the field may thus be regarded approximately as a superposition of free waves. The field can then be represented as a Fourier-Stieltjes integral (which we write in a more convenient sum notation)

\[ \zeta(x, t) = \sum_k \left[ \eta_k^* e^{i(k \cdot x - \sigma t)} + \eta_k e^{-i(k \cdot x - \sigma t)} \right] \]  

(2.1.1)

where \( \zeta \) is the surface displacement (positive upwards),

\( x = (x_1, x_2) \) is the horizontal coordinate vector,

\( k = (k_1, k_2) \) is the horizontal wave-number vector,

\( \sigma = (gk \tanh kH)^{1/2} \) is the frequency of a free surface gravity wave in water of depth \( H \), which we take to be constant, and

\( \eta_k \) is a random Fourier amplitude.
For a homogeneous, stationary wave field, the covariance matrix of the Fourier amplitudes is diagonal,

\[
\langle \eta_{k_1} \eta_{k_2} \rangle = \begin{cases} 
0, & \text{for } k_1 = k_2 \\
0, & \text{for } k_1 \neq k_2 
\end{cases}
\]

\[
\langle \eta_{k_1} \eta_{k_2}^* \rangle = \frac{2}{\rho g} F(k) \Delta k
\]

where the cornered parentheses denote ensemble means, \( \rho \) is the density of water, \( g \) is the acceleration of gravity, \( \Delta k \) is the wave-number increment of the Fourier sum, and \( F(k) \) is the (continuous) energy spectrum.

The total wave energy per unit surface area is then

\[
E = \frac{1}{2} g \langle \xi^2 \rangle = 2 \int F(k) \Delta k
\]

The normalisation of \( F \) is that used in the general interaction theory; it differs from the more usual definition by a factor \( 2/\rho g \).

It can be shown that a homogeneous field consisting only of dispersive, free-wave components rapidly attains a Gaussian state (Hasselmann, 1967a). In this case the energy spectrum \( F(k) \) completely specifies the field statistically.

Since the wave components undergo weak interactions, the Gaussian property, the stationarity and the homogeneity of the wave field are only approximately valid. The field can still be described locally by a spectrum, but this must now be regarded as a slowly varying function of \( x \) and \( t \), where

\[
\frac{1}{k} \frac{\partial F}{\partial x} \ll 1, \quad \frac{1}{\sigma t} \frac{\partial F}{\partial t} \ll 1.
\]