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ON SECOND ORDER CONTRIBUTIONS TO SHIP WAVES
AND WAVE RESISTANCE

von

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The O.N.R. - N.S.F. Symposium on Wave Resistance Theory in Ann Arbor held 1963 made clear that current research is focussed around the following items:

(A) Determination of quantities from the wave pattern representative for wave resistance.
(B) Formal and semi-empirical corrections to the classical linearized theory.
(C) More refined techniques for optimizing ship forms within linear theory.

Within the present paper I shall report on work done since then which might provide material to enforce progress in any of these directions. The "pièce de résistance" of this contribution is the gradual evolution of a computer program which in a rationalized way gives the basic information of flow and wave components due to typical singularities, - (as discrete doublets, doublet struts, continuous parabolic distributions on submerged lines, infinite and truncated vertical planes) - all within linearized approach. This information lends itself readily for application to item (A). Any method proposed for determination of energy flow from characteristics of the flow, in peculiar from the geometry of wave pattern, can be tested for accuracy and for consistency on such a theoretical wave field available numerically before entering into expensive experimental work which provides in general too little reliable information on optimal choice of region where to perform measurements. The overwhelming part of the methods proposed for (A) is implicitly based on validity of certain asymptotic representations for the wave pattern. Only numerical calculations can tell what distances are already large enough, especially regarding decay of the so called "local flow components" in order that such representations may be applied.
For (B), the theories of wave resistance used nowadays are of second order, based on linearized flow models. For calculation of wave resistance, only a far field component of this flow has to be known explicitely; for any consistent approach to third order resistance contributions, however, the knowledge of the entire first order flow is essential. Aside from some semi-empirical approaches to alternate formulations of linear theory, which we shall submit to some critical examen, and aside from indirect approach as successfully carried out by Kajitani recently, the tool for a systematic perturbation attack to the higher order flow components has been provided by Wehausen in a series of papers starting with that read before this audience in 1956 up to his contribution to the Ann Arbor conference. As, however, the step to formulate resistance expressions was not performed, credit is generally given to Sisov for first dealing with these. We should, nevertheless, be aware, that expressions given by Sisov so far essentially contain divergent integrals due to selection of improper radiation condition for Green's function of pressure point. In our present investigation, we will rederive some of Sisov's results from a Green's theorem approach essentially following Wehausen. We will, in particular, show up some simplifications which make calculations straightforward once a Fourier representation of first order flow components is given. It will become evident that integration over undisturbed free surface has to be performed only in a small domain where local flow is significant; third order wave resistance is, therefore, much more tractable to numerical evaluation than is apparent from what was formulated by Sisov, provided we decide on an appropriate definition of wave resistance.

We decided to deviate from Wehausen's approach by some simplifications regarding the actual flow boundaries. However, the resulting expressions found for third order resistance depend in a simple manner only on ship's offsets and on first order velocity components. We, therefore, feel that these deviations at least have not introduced artificial complications against results still to be found from more refined analysis.

Regarding the third problem, i.e. ships of minimum resistance within lowest order theory, our investigation should throw light on the question to what degree third order contributions might counteract the tendencies predicted. At the present stage, however, our calculations are limited to a two-parameter class of hull forms having parabolic waterlines. This is mainly due to the fact that we prefered analytical evaluation of integrals over the geometry of the ship. An extension of our programm for local flow, to include contributions from
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Empirical surface elements is feasible, but loss of closed integration would probably increase time for computation and weaken control of accuracy. Moreover, the necessary degree of hull-subdivision will in general depend on Froude number and is not known beforehand. Even for analytical ship forms, the development of formal expressions for closed integration cannot be done by the computer and provides many opportunities for errors in evaluation of singular regions of integrands for local flow.

Description of analysis to derive potential and wave resistance.

We shall essentially follow the approach of Wehausen [3], but modify it for flow in a tank of rectangular cross section. This will simplify the formulation of radiation conditions for the flow and allows the use of a Green's function in a Fourier series representation regarding the ordinate \( y \) chosen in direction perpendicular to the vertical tank walls. The ship's motion is in the +x-direction with speed \( c \), the \( z \) coordinate is taken vertically upwards to conform with earlier work [6]. As far as possible we otherwise use notation consistent with [3]. However, direction of normal vectors is reversed resulting from our definition of Green's function with an opposite sign. Extension of results to unrestricted water is straightforward.

1. Derivation of second order potential.

We introduce dimensionless coordinates as \( X = 2x/L \), \( Y = 2y/L \), \( Z = 2z/L \), where \( L \) is the ship's length. The velocity potential is nondimensionalized as \( \phi = 2 \phi/Lc \).

As speed parameter we use \( \mathbf{r} \), \( \sqrt{2g/L/2c} \).

Let \( Y = t F(X, Z) \) be the dimensionless representation of the hull geometry, where \( B \) is the ship's breadth. \( c \approx B/L \) will serve as a perturbation parameter and is considered as a small quantity. Let \( X = X_a \) and \( X = X_e \) be the equations of two vertical control planes \( S_a \) and \( S_e \) ahead of and behind the ship. Let \( S_b \) stand for the tank bottom plane \( Z = -H \). Let \( Y = T \) be the equations of the vertical tank walls \( S_r \) and \( S_l \), where \( T = b/L \), \( b = \) tank width. Let \( S_f \) stand for the free surface \( Z = \psi(X, Y) \) for \( X_e < X < X_a, T < Y < T \); let \( S^*_f \) stand for the undisturbed free surface \( Z = 0 \) with the waterplane area of the ship excluded. Let \( S_w \) stand for the wetted surface of the ship and \( S^*_w \) stand for the part of the surface up to \( Z = 0 \). Let \( D \) stand for the domain of the complete flow, bounded by \( S^*_w, S_f, S_a, S_e, S_r, S_l \) and \( S_b \). Let \( \psi^\infty \) describe the corresponding domain if \( S_w \) and \( S_f \) are replaced by \( S^*_w \) and \( S^*_f \). Let \( \psi'' \) stand for the Michell type first approximation to the exact potential \( \psi \), let \( P \) stand for a point
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in D or D° with coordinates X, Y, Z and let P' represent a point on a boundary surface with coordinates ξ, η and ζ. Let G(P, P') stand for the potential of a source of output 4π as defined in the Appendix.

The functions ψ, ψ'' and G of the variables X, Y, Z are subject to the following set of conditions:

A. Laplace equation: Δψ = 0 in D, Δψ'' = 0 in D°,
   ΔG = 4πδ(P - P') in D°
   where Δ stands for δ²/δx² + δ²/δy² + δ²/δz² and δ means the Dirac delta function, which is zero if P is unequal P'; (condition A. implies that G becomes singular as -1/|P - P'|).

B. On S° we have (linearized free surface condition):
   \( γ_0 ψ'' + ψ''_{xx} = 0 \)
   \( γ_0 G_z + G_{xx} = 0 \)

For the exact potential ψ, no such conditions holds. But we define a function δ(X, Y) by

\( γ_0 ψ_z + ψ_{xx} = δ(X, Y) \).

C. On S₁ and S₀ we have: \( ψ_y = 0 \); \( ψ''_y = 0 \); \( G_y = 0 \).

D. On S₂ we have: \( ψ_z = 0 \); \( ψ''_x = 0 \); \( G_z = 0 \).

E. On S₃ we have:
   \( ψ_n = ±εF_X/\sqrt{ε^2F_X^2 + ε^2F_Z^2 + 1} \)
   where ± stands for positive or negative and the index n stands for derivation in normal direction out of the fluid's domain D.
   On S₃°, the projection of S₃° on the plane Y = 0, we have \( ψ''_y = ±εF_X \) for the first order potential.

F. For fixed P' we have:
   \( G = O(1) \) with \( X \rightarrow -∞ \);
   \( G = O(X^{-1}) \) with \( X \rightarrow +∞ \);
   \( G_z = O(1) \) with \( X \rightarrow -∞ \);
   \( G_x = O(X^{-1}) \) with \( X \rightarrow +∞ \).