On an Instability of Surface Waves.

M. Schuler\(^1\) has described a sudden change of the wave pattern occurring in the neighbourhood of a wave generator. Under certain conditions, suddenly transverse waves appear the period of which is double the period of the progressive waves. If a plunging sphere is used as generator, under certain conditions waves with radial (instead of circular) crests appear.

As until now a theoretical foundation of this phenomenon has not yet been given; we have tried to find out at least some of the conditions necessary for a sudden change of that kind.

The apparent gravity of any surface particle in a progressive wave of amplitude \(a\) and frequency \(\omega/2\pi\) is the sum of gravity \(g = 981 \text{ cm/s}^2\) and kinematic acceleration \(\ddot{h}(t) = -a \omega^2 \sin t\).

\[
\ddot{h}(t) = \ddot{g} + \ddot{b}(t) = g \cdot (1 + \beta \omega^2 t).
\]

(1)

The free surface condition for a superimposed flow with potential \(\phi\) is thus

\[
\frac{\partial^2 \phi}{\partial t^2} + (1 + \beta \omega^2 t) \cdot g \cdot \frac{\partial \phi}{\partial t} = 0.
\]

(2)

\(z\) being the vertical coordinate. Now if

\[
\phi(x,y,t) = e^{-ka} \cdot \cos ky \cdot A(t)
\]

is the potential of a standing transverse wave in a cross section \(0 \leq y \leq B\) of the tank of breadth \(B\),

\(1\) M. Schuler, "Der Umschlag von Oberflächenwellen". ZAMM 13 (1933) 443 – 446.