On a Simple Method for Calculating Laminar Boundary Layers

by

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Summary: A simple one parametric method, due to A. Walz and based on the momentum and energy equations, for calculating approximately laminar boundary layers is extended to cover axi-symmetric flow as well as plane flow. The necessary computing work is reduced a little.

Another known method which requires still less computing work is also extended for axi-symmetric flow and, with the amendment of a numerical constant, proves adequate for practical purposes.

1. Introduction

Since there are already several methods of calculating approximately laminar boundary layers for incompressible flow, it would seem necessary to justify the development of yet another method. The following method tries to obtain in a practical manner, results as correct as possible and with the smallest amount of computing work. Obviously a very accurate method is to represent the velocity profiles approximately by a two parametric class whereby the partial differential equation for boundary layers is replaced by two ordinary differential equations: the momentum and the energy equation\(^1\). But here the interpolation in the two parameters requires considerably more computing work than that with the usual one parametric methods. Therefore A. Walz\(^2\) altered this method back again into a one parametric method using the energy equation instead of the boundary condition for the second derivative \(\partial^2 u / \partial y^2\) on the wall. By this means he obtained better results than by the usual one parametric methods based on the momentum equation only. So he proved that it is more important to satisfy the energy balance on the average over the whole layer rather than the usual boundary condition for the curvature of the velocity profile at the wall. As the latter condition is too sharp for a limited variety of profiles, a wrong profile is sometimes selected by it. The method of A. Walz can be altered slightly as follows. It is extended so as to calculate boundary layers on axi-symmetric bodies (without incidence) as well as in two-dimensional plane flow. Also, the computing work is reduced.

Notation

\[x\] the axial co-ordinate
\[r\] the radial co-ordinate
\[r_0\] the co-ordinate of the body of revolution

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s the distance measured along the surface of the body
n the distance normal to the surface
\[
\frac{r}{r_0} = 1 + \frac{r_0}{n \cos \alpha} \text{ with } \tan \alpha = \frac{dr_0}{dx}
\]
u, \nu the normal velocity components at any point in the layer
U the velocity at the edge of the boundary layer
\[U_0\] the velocity of the undisturbed stream
ρ the density of the incompressible fluid
μ the viscosity
\[\nu = \mu / \rho = \text{the kinematic viscosity}\]
δ the thickness of the boundary layer
\[
\delta_1 = \int_0^\infty \left(1 - \frac{u}{U}\right) \frac{r}{r_0} dn = \text{the displacement thickness}
\]
\[
\delta_2 = \int_0^\infty \frac{u}{U} \left(1 - \frac{u}{U}\right) \frac{r}{r_0} dn = \text{the momentum thickness}
\]
\[
\delta_3 = \int_0^\infty \frac{u}{U} \left[1 - \left(\frac{u}{U}\right)^2\right] \frac{r}{r_0} dn = \text{the energy thickness}
\]
\[D = \int_0^\infty \left(\frac{\partial (u|U)}{\partial (n/\delta)}\right)^2 \frac{r}{r_0} d\left(\frac{n}{\delta_2}\right) = \text{the dimensionless dissipation in the layer}
\]
\[H = \delta_1 / \delta_2 = \text{a parameter characterising the velocity profile}
\]
\[H_{12} = \delta_3 / \delta_2
\]
\[R_2 = U \delta_2 / \nu = \text{the local Reynolds number of the boundary layer}
\]
\[R = U_0 R' / \nu = \text{the Reynolds number of the main stream}
\]
\[R' = \text{a characteristic length}
\]
\[X = \delta_2 R_2 = U \delta_2^2 / \nu = \text{a useful computing quantity}
\]
\[\tau_0 = \mu \left(\frac{\partial u}{\partial n}\right)_{n=0} = \text{the shear stress at the wall = the skin friction}
\]
\[\varepsilon = \left(\frac{\partial (u|U)}{\partial (n/\delta)}\right)_o = \frac{\tau_0}{\rho U^2 R_2} = \text{the dimensionless skin friction}
\]
\[\lambda = -\frac{\delta_2^2}{U} \left(\frac{\partial^2 u}{\partial n^2}\right)_o = \text{another characteristic profile parameter}
\]