Philip Herbert Augener

COMPUTATION OF WAVE DRIFT FORCES FOR DYNAMIC POSITIONING WITHIN THE EARLY SHIP DESIGN
COMPUTATION OF WAVE DRIFT FORCES FOR DYNAMIC POSITIONING WITHIN THE EARLY SHIP DESIGN

Vom Promotionsausschuss der Technischen Universität Hamburg-Harburg zur Erlangung des akademischen Grades Doktor-Ingenieur genehmigte Dissertation

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aus
Essen

2016
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Tag der mündlichen Prüfung:
22.06.2016

© Schriftenreihe Schiffbau der
Technischen Universität Hamburg-Harburg
Am Schwarzenberg-Campus 4 (C)
21073 Hamburg
https://www.tuhh.de/vss/

Bericht Nr. 693
ISBN 978-3-89220-693-4
Acknowledgment

In the very first place I would like to thank my doctoral thesis supervisor Prof. Dr.-Ing. Stefan Krüger for giving me the opportunity to write this dissertation. The support during this thesis was excellent and working at the Institute of Ship Design and Ship Safety at the Hamburg University of Technology was great.

In second place I would like to express my gratitude to Prof. Dr.-Ing. Moustafa Abdel-Maksoud for his willingness to be my second supervisor.

In third place I am grateful to Prof. Dr.-Ing. Wolfgang Fricke for taking over the chairmanship of my doctoral examination procedure as well as being an additional supervisor. Furthermore I would like to thank Prof. Dr.-Ing. Friedrich Wirz for being an additional supervisor of this thesis as well.

This work is part of the research project DYPOS, which is funded by the Federal Ministry of Economics and Technology of Germany. I really appreciate this funding. Moreover I am grateful for the provision of the wave basin data by the J. J. SIETAS KG. By name special thanks go to Dr.-Ing. Hendrik Vorhölter for making the validation of my computation procedure possible.

Further thanks go to my colleagues at the institute for the many discussions not only concerning naval architecture. Especially I want to express my thankfulness to Adele Lübcke, M.Sc., for her support during the finalization of this thesis. In addition I would like to thank Dr.-Ing. Hendrik Dankowski for the convenient working atmosphere in our office.

Last but not least I would like to thank my family and friends for the really great backup during the last years. In particular I would like to thank my parents, who supported me by continuously emphasizing that the effort is worth it.
Abstract

In order to fulfill the requirements of the German offshore wind industry individually designed and constructed offshore wind farm installation vessels are needed. One of the primary design constraints of these ships is dynamic positioning (DP). For the correct design of DP systems static and dynamic calculations should already been performed during the early ship design. Among other input data time series and time averaged mean values of the horizontal wave drift forces as well as the wave drift yaw moment are needed. For this a calculation procedure based on strip theory in combination with a correction for short waves is presented and validated with experimental results for the SIETAS TYPE 187.
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Nomenclature

\begin{itemize}
  \item \(L\) \hspace{1em} Length of the vessel
  \item \(T\) \hspace{1em} Draft of the vessel
  \item \(m\) \hspace{1em} Total mass of the vessel
  \item \(g\) \hspace{1em} Gravitational acceleration
  \item \(\Delta t\) \hspace{1em} Time step
  \item \(\Theta\) \hspace{1em} 3x3 matrix of inertia with respect to COG of the vessel
  \item \(\beta\) \hspace{1em} Wave propagation direction \(\beta = \pi - \mu\)
  \item \(\gamma\) \hspace{1em} Waterline angle regarding the longitudinal axis of the vessel
  \item \(\omega\) \hspace{1em} Circular wave frequency
  \item \(\rho\) \hspace{1em} Density of sea water
  \item \(\mu\) \hspace{1em} Wave encounter angle
  \item \(L_1\) \hspace{1em} Non-shadow part of the waterline
  \item \(T_p\) \hspace{1em} Peak period
  \item \(T_1\) \hspace{1em} Average wave period
  \item \(x_g\) \hspace{1em} Longitudinal coordinate of the center of gravity
  \item \(y_g\) \hspace{1em} Transverse coordinate of the center of gravity
  \item \(x_0\) \hspace{1em} Longitudinal coordinate of a waterline element
  \item \(y_0\) \hspace{1em} Transverse coordinate of a waterline element
  \item \(\hat{p}_w\) \hspace{1em} Complex amplitude of the pressure at a point on the mean waterline
  \item \(\vec{x}\) \hspace{1em} Position vector of a waterline element
  \item \(\hat{\alpha}\) \hspace{1em} Complex amplitude of the vessel's rotation vector
  \item \(\vec{x}_G\) \hspace{1em} Vector of center of gravity
  \item \(\zeta_a\) \hspace{1em} Real wave amplitude of an elementary wave
  \item \(\hat{\zeta}\) \hspace{1em} Complex wave amplitude
  \item \(\zeta\) \hspace{1em} Real wave amplitude
  \item \(\mu_0\) \hspace{1em} Main wave propagation direction
  \item \(\omega_c\) \hspace{1em} Encounter frequency between the ship and the waves
  \item \(\omega_0\) \hspace{1em} Modal frequency of the specified seaway class
  \item \(\epsilon_j\) \hspace{1em} Random phase angle of irregular waves
  \item \(\epsilon_{jl}\) \hspace{1em} Random phase angle of natural seaway
  \item \(\hat{Y}_p\) \hspace{1em} Complex transfer function of the pressure at a point on the mean waterline
  \item \(\hat{Y}_4\) \hspace{1em} Complex transfer function of the roll motion
  \item \(\hat{Y}_5\) \hspace{1em} Complex transfer function of the pitch motion
  \item \(\Delta_s\) \hspace{1em} Directional vector of each waterline element
  \item \(A_{x0}\) \hspace{1em} Sectional area of the submerged transom in still water
  \item \(H_{1/3}\) \hspace{1em} Significant wave height
\end{itemize}
Nomenclature

$z_{x0}$ Vertical coordinate of the center of area of the submerged transom in still water

$\hat{Y}_{zr}$ Complex transfer function of the relative vertical motion between the ship and the water surface at the waterline of the ship

$\hat{Y}_{1s}$ Complex transfer function of the surge motion of the ship’s center of gravity

$\hat{Y}_{2s}$ Complex transfer function of the sway motion of the ship’s center of gravity

$\hat{Y}_{3s}$ Complex transfer function of the heave motion of the ship’s center of gravity

$\hat{Y}_{5}^*$ Conjugate-complex transfer function of the pitch motion

$\hat{Y}_{6}^*$ Conjugate-complex transfer function of the yaw motion

$y_w^+$ Absolute value of the transverse coordinate of the waterline

$\alpha^2$ Drift force coefficient

$\alpha_{0}^2$ Mean drift force coefficient

$S_{zz}$ Energy density spectrum of a seaway

$F^2(t)$ Wave drift force or wave drift moment of yaw in the time domain

$F^2_{\xi}(t)$ Longitudinal wave drift force in the time domain

$F^2_{\eta}(t)$ Transverse wave drift force in the time domain

$M^2_{\zeta}(t)$ Wave drift moment of yaw in the time domain

$a^2(t)$ Square of the envelope of the random seaway

$\frac{dy^+}{dx}$ Inclination of the waterline against the longitudinal axis

$\hat{Y}_{zr0}$ Complex transfer function of the relative vertical motion between the ship and the water surface at the centerline of the ship

$\hat{Y}_{5W}^*$ Conjugate complex amplitude of the transverse slope of the water surface

$\overline{F^2_{\xi}}$ Time averaged longitudinal wave drift force in the frequency domain

$\overline{F^2_{\eta}}$ Time averaged transverse wave drift force in the frequency domain

$\overline{M^2_{\zeta}}$ Time averaged wave drift moment of yaw in the frequency domain

$\hat{\alpha}^*$ Conjugate complex amplitude of the vessel’s rotation vector
1. Introduction

1.1. The Importance of the Early Ship Design Phase

Basically the European shipbuilding industry still exists because of its competence in the fast design and successful building of tailor-made ships in small numbers. The times when large series of the same vessel were built in Europe are over. However, due to the competence in building one of a kind ships as well as very short series with economic success, there is still a number of existing shipyards in Europe. These shipyards are all very specialized and each one is able to design a ship extremely fast. Figure 1 clearly shows the importance of the early design stage for the economic success of the whole project. 70% of the cost of a ship are fixed within the first four weeks. Thus the early design stage is essential for the profit and risk level of a shipbuilding project.

Figure 1: The Cost Impact of the Early Ship Design (Krüger, 2003)

To be able to design a tailor-made vessel within a couple of weeks, fast and reliable tools for the early ship design are required. These tools have to enable the naval architect to do all kinds of relevant calculations fast and accurately for the initial design of the new project. This can be ensured by the use of so-called first principle based methods. These direct calculation methods can deliver results quickly and may be less conservative than rule based design and empirical design formulae, which directly increases the competitiveness of the shipyard. This is achieved by the accurate modeling of the relevant physics and the use of detailed input data, such as the actual hull form, compartmentation and...
weight distribution. A number of first principle computation methods can, for example, be found in the ship design environment E4 as described in Bühr et al. (1994) and Todd (2002).

1.2. Station Keeping

The capability of a vessel to keep its position and heading is called station keeping. While this is well established in marine engineering, e.g. in the offshore oil and gas industry, it is of increasing importance in conventional ship design. Depending on the application, station keeping in marine engineering can be achieved either by fixed mooring systems or so-called dynamic positioning systems. Contrary to that, the use of fixed mooring systems is not applicable in the shipbuilding industry. This is caused by the typical transportation problems of ships, which in general do not allow the time-consuming installation of fixed mooring systems. Therefore, dynamic positioning (DP) is the preferred solution for station keeping in shipbuilding and it is an increasing task for different ship types.

Examples of the application of dynamic positioning in shipbuilding are mega yachts and cruise liners, which need DP systems in order to avoid environmental damages due to their anchoring operations. This very special case of dynamic positioning is also called virtual anchoring. Another very important application of dynamic positioning systems in shipbuilding are all kinds of offshore supply activities as well as offshore crane operations. Two examples of this are platform supply vessels on the one hand and the increasing number of heavy-lift jack-up vessels for the transportation and erection of offshore wind turbines on the other hand.

1.3. The German Offshore Wind Industry

The Federal Government of Germany has agreed upon the so-called turnaround for sustainability, namely the changeover from the fossil and nuclear based electrical power production to renewable energies. One major stakeholder of this turnover is the offshore wind industry with its offshore wind farms. Each wind farm consists of a large number of offshore wind turbines. These wind turbines will be erected in the exclusive economic zone (EEZ) of Germany. The offshore wind farm projects in the German EEZ in the area of the North Sea are shown in Fig. 2 and the ones in the Baltic Sea in Fig. 3. The number of wind farm projects in the North Sea is larger than in the Baltic Sea. Furthermore, the figures clearly illustrate the large distances from the coastline to the erection sites, in
particular in the North Sea. These large distances are due to the fact that the offshore wind turbines in Germany should not be visible from ashore.

![Map of Offshore Wind Farms in the German EEZ](image1.png)

**Figure 2: Offshore Wind Farms in the German EEZ in the North Sea (BSH, 2014d)**

![Map of Offshore Wind Farms in the German EEZ in the Baltic Sea](image2.png)

**Figure 3: Offshore Wind Farms in the German EEZ in the Baltic Sea (BSH, 2014b)**

In addition to longer transportation times for the offshore wind turbines and foundations, the great distances to the coastline directly lead to large water depths as shown in Figs. 4 and 5. A good example of this is the German offshore wind farm "Global Tech 1" as mentioned in Wehrmann (2012). The distance from its offshore-base port Bremerhaven...
to this wind farm is about 100 nautical miles and the water depth at the installation site is approximately 40 m.

Figure 4: Water Depths in the German EEZ in the North Sea (BSH, 2014c)

Figure 5: Water Depths in the German EEZ in the Baltic Sea (BSH, 2014a)
1.4. Offshore Wind Farm Installation Vessels

1.4.1. Generations of Offshore Wind Farm Installation Vessels

The beginning of the commercial offshore wind industry is located in the shallow coastal waters of Denmark and the United Kingdom. For the erection of the local wind farms simple jack-up platforms have been used. These platforms are primarily designed for harbor construction and the offshore oil and gas industry. Even though some of them have dynamic positioning systems, most of these platforms still do not have strong propulsion systems and hence have to be towed to the erection sites. Furthermore, these platforms only have limited payload and crane capacities. According to Hochhaus (2012a) these vessels are called the first generation of offshore wind farms installation vessels.

In order to be able to build offshore wind farms at larger distances from the coast and correspondingly in greater water depths, existing cargo vessels have been converted to offshore wind farm installation vessels. The vessels are bigger than the first generation, self-propelled and sometimes additionally retrofitted with jack-up systems. However, they still only have limited crane capacities of up to 400 tons. Since these vessels are only converted for their new function, it is always a trade-off. Nowadays vessels are called the second generation of offshore wind farm installation vessels.

During the planning of the offshore wind farms in the German EEZ it became obvious that the large distances from the shore as well as the great water depths lead to harsh weather conditions at the erection sites of the offshore wind farms, especially in the North Sea. In addition, the size and weight of the offshore wind turbines are growing and with it the foundations as well. Furthermore, in order to fulfill the ambitious goals of the German Government, thousands of offshore wind turbine generators have to be installed. Since the wind turbines and their foundations are not erected in one piece, an even higher number of offshore heavy-lift crane operations is required. Following on from this and supported by the general cost pressure for ship owners, it is definitely not possible to wait for calm environmental conditions for the installation. This is a principle difference of the offshore wind industry to the offshore oil and gas industry, where it is common practice to wait with an offshore installation until the weather is calm. Therefore, the offshore wind turbine installation vessels for the German offshore wind industry have to be able to install offshore wind turbines even during harsh weather conditions. In order to achieve this ambitious goal, individually designed and constructed vessels are needed, since the wind farm installation vessels of the first and second generation are technically and economically not able to fulfill this new industrial mission. These purpose-built ships are called the third generation of offshore wind farm installation vessels.
1.4.2. Design Boundary Conditions for the Third Generation of Offshore Wind Farm Installation Vessels

The industrial mission of the third generation of offshore wind farm installation vessels directly leads to a number of boundary conditions for the ship design. These boundary conditions have to be satisfied in an optimal way in order to make the vessel fulfill the specific requirements of the German offshore wind industry. In the following the major design boundary conditions for this specific ship type are itemized:

- limited main dimensions to enable access to a variety of offshore-base ports
- relatively high velocities for short transportation and transit times between the ports and the offshore wind farms
- large deck areas for the flexible stowage of foundations, wind turbines and other required equipment
- cargo deck strengthened for heavy cargo for the flexible stowage of foundations, wind turbines and other required equipment
- heavy-lift crane capacity for the offshore installation of state-of-the-art offshore wind turbines
- jack-up capability for safe crane operations with only a minimum impact of the seaway in order to minimize down times of the vessel
- powerful dynamic positioning systems for safe jack-up operations even during harsh weather conditions in order to minimize down times of the vessel
- accommodation for a large number of crew and special personnel for the erection of wind farms
- long jack-up legs to enable jack-up operations even in large water depths
- satisfactory payload capacity in order to transport a reasonable number of required offshore equipment at once
- crane around one leg arrangement in order to maximize the free deck space

1.4.3. MV INNOVATION

A very good example of an offshore wind farm installation vessel of the third generation is the MV INNOVATION. This heavy-lift jack-up vessel was built in Poland and it was
delivered in 2012. According to Wehrmann (2011) this vessel was the most powerful off-shore wind farm installation vessel on its completion. A summary of the main data is given in Table 1. For a better understanding of the ship there is an illustration of the front view given in Fig. 6 and of the top view in Fig. 7.

Table 1: Main Data of the MV INNOVATION based on (HGO, 2014) and (Hochhaus, 2012b)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length overall (hull)</td>
<td>147.50 [m]</td>
</tr>
<tr>
<td>Breadth (hull)</td>
<td>42.00 [m]</td>
</tr>
<tr>
<td>Draft (design)</td>
<td>7.00 [m]</td>
</tr>
<tr>
<td>Depth to Main Deck</td>
<td>11.00 [m]</td>
</tr>
<tr>
<td>Propulsion system</td>
<td>diesel-electric</td>
</tr>
<tr>
<td>Main engines</td>
<td>6 x 4,500 [kW]</td>
</tr>
<tr>
<td></td>
<td>1 x 1,620 [kW]</td>
</tr>
<tr>
<td>Azimuth thrusters (stern)</td>
<td>4 x 3,500 [kW]</td>
</tr>
<tr>
<td>Tunnel thrusters (bow)</td>
<td>3 x 2,800 [kW]</td>
</tr>
<tr>
<td>Deadweight (design)</td>
<td>9,323 [t]</td>
</tr>
<tr>
<td>Speed (design)</td>
<td>12.00 [kts]</td>
</tr>
<tr>
<td>Crane type main hoist</td>
<td>crane around one leg</td>
</tr>
<tr>
<td>Crane lift main hoist (jacked up) (31.5 m outreach)</td>
<td>1,500 [t]</td>
</tr>
<tr>
<td>Crane lift main hoist (aft) (20 m outreach)</td>
<td>1,250 [t]</td>
</tr>
<tr>
<td>DP class</td>
<td>DP 2</td>
</tr>
<tr>
<td>Leg length</td>
<td>90.00 [m]</td>
</tr>
<tr>
<td>Clear deck area</td>
<td>3,400 [m²]</td>
</tr>
<tr>
<td>Maximum specific deck load</td>
<td>15.00 [t/m²]</td>
</tr>
<tr>
<td>Accommodation</td>
<td>100 [persons]</td>
</tr>
<tr>
<td>Length/Breadth</td>
<td>3.51</td>
</tr>
<tr>
<td>Breadth/Draft</td>
<td>6.00</td>
</tr>
<tr>
<td>$C_B$ (design)</td>
<td>0.76</td>
</tr>
<tr>
<td>$C_M$ (design)</td>
<td>0.99</td>
</tr>
</tbody>
</table>

The vessel is equipped with a powerful jack-up system with long legs and a heavy-lift crane, which allows safe heavy-lift crane operations in deep water and during harsh weather conditions. The large deck area in combination with the crane around one leg arrangement as well as the high payload capacity permits a flexible stowage of all kinds of equipment required for the installation. The accommodation is designed for a large number of crew and special personal and the diesel-electric propulsion system, consisting of four azimuth thrusters at the stern and three transverse tunnel thrusters at the bow,
enables rather high velocities as well as a very good dynamic positioning capacity. It can be stated that the MV INNOVATION fulfills the design boundary conditions for the third generation of offshore wind farm installation vessels very well. Resulting from the optimum solution for the industrial mission, the length/breadth ratio is very small and the breadth/draft ratio as well as the block ($C_B$) and main section ($C_M$) coefficients are very high in comparison to conventional cargo ships.

Figure 6: Front View of the MV INNOVATION (RINA, 2012)

Figure 7: Top View of the MV INNOVATION (RINA, 2012)
1.4.4. SIETAS TYPE 187

Another example of an offshore wind farm installation vessel of the third generation is the SIETAS TYPE 187. A good overview of the development of this vessel can be found in Vorhölter et al. (2012). This vessel is the first German designed and constructed ship of this kind. Figure 8 shows the profile view of the vessel and in Fig. 9 the top view is shown. The main characteristics of this vessel can be found in Tab. 2.

Even though the SIETAS TYPE 187 is a little bit smaller than the MV INNOVATION, the vessel achieves the optimal solution of the boundary conditions in the same good way. A major difference between the two vessels is the propulsion and maneuvering system. Contrary to the MV INNOVATION the SIETAS TYPE 187 is equipped with two controllable pitch propellers mounted on conventional shaft lines. In addition, two transverse stern and two bow thrusters as well as two high-lift rudders are installed. Following from the boundary conditions the length/breadth ratio is small and the breadth/draft ratio as well as the block ($C_B$) and main section ($C_M$) coefficients are high as well.

Figure 8: Profile View of SIETAS TYPE 187
1 INTRODUCTION

Figure 9: Top View of SIETAS TYPE 187

Table 2: Main Data of SIETAS TYPE 187

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length overall (hull)</td>
<td>m</td>
<td>139.40</td>
</tr>
<tr>
<td>Breadth (hull)</td>
<td>m</td>
<td>38.00</td>
</tr>
<tr>
<td>Draft (design)</td>
<td>m</td>
<td>5.70</td>
</tr>
<tr>
<td>Depth to Main Deck</td>
<td>m</td>
<td>9.12</td>
</tr>
<tr>
<td>Propulsion system</td>
<td></td>
<td>diesel-electric</td>
</tr>
<tr>
<td>Main Engines</td>
<td>kW</td>
<td>4 x 4,500</td>
</tr>
<tr>
<td>Propulsion</td>
<td>kW</td>
<td>10,000</td>
</tr>
<tr>
<td>Bow thrusters</td>
<td>kW</td>
<td>2 x 2,500</td>
</tr>
<tr>
<td>Stern thrusters</td>
<td>kW</td>
<td>2 x 2,500</td>
</tr>
<tr>
<td>Deadweight (design)</td>
<td>t</td>
<td>6,500</td>
</tr>
<tr>
<td>Speed (at 5.4 m draft)</td>
<td>kts</td>
<td>12.00</td>
</tr>
<tr>
<td>Crane type main hoist</td>
<td></td>
<td>crane around one leg</td>
</tr>
<tr>
<td>Crane lift main hoist (30 m outreach)</td>
<td>t</td>
<td>900</td>
</tr>
<tr>
<td>DP class</td>
<td></td>
<td>DP 2</td>
</tr>
<tr>
<td>Leg length</td>
<td>m</td>
<td>81.00</td>
</tr>
<tr>
<td>Clear deck area</td>
<td>m²</td>
<td>3,300</td>
</tr>
<tr>
<td>Maximum specific deck load</td>
<td>t/m²</td>
<td>10.00</td>
</tr>
<tr>
<td>Accommodation</td>
<td>persons</td>
<td>74</td>
</tr>
<tr>
<td>Length/Breadth</td>
<td></td>
<td>3.67</td>
</tr>
<tr>
<td>Breadth/Draft</td>
<td></td>
<td>6.67</td>
</tr>
<tr>
<td>$C_B$ (design)</td>
<td></td>
<td>0.82</td>
</tr>
<tr>
<td>$C_M$ (design)</td>
<td></td>
<td>0.99</td>
</tr>
</tbody>
</table>

There are various reasons why the SIETAS TYPE 187 has been chosen as the validation vessel for this thesis. First of all, the very kind provision of the wave basin data for this vessel by the shipyard has to be mentioned. Furthermore, the blunt, flat and wide hull shape makes the ship interesting. Additionally, the currently high interest in this new
ship type as well as the importance of dynamic positioning for this ship made the vessel suitable for this thesis.

1.5. Conclusions

The general importance of the early ship design for the success of a project is highlighted and first principle based methods are pointed out to be very suitable for the early ship design. Subsequently, the characteristics of the Germany offshore wind industry are emphasized and from this the need for purpose-built offshore wind farm installation vessels is derived, leading to a set of design boundary conditions. Two examples of offshore wind farm installation vessels of the third generation are presented which fulfill these design boundary conditions very well leading to blunt, flat and wide vessels. From these combinations of main dimensions, the naval architect has to deal with an increased number of hydrodynamic problems and the use of well-established methodologies from naval architecture have to be questioned. The importance of dynamic positioning as a major design task is identified for these ships and thus this rather new ship type is a very good application for the methods developed during this thesis.
2. Dynamic Positioning within the Early Ship Design

2.1. Introduction and State of the Art

Dynamic positioning is the ability of a vessel to keep its position and heading by using the installed propulsion and maneuvering system to counteract the environmental forces acting on the ship. During DP operations, defined deviations of the position and the heading are permitted. Based on this, a DP system has to be designed in a way to keep the vessel within the specified position and heading limits as well as to minimize fuel consumption and wear and tear on the propulsion equipment. In addition, different failure modes of parts of the propulsion and maneuvering system have to be considered.

The advantages of a vessel equipped with a DP system are the avoidance of tugs, fast reaction times for changes of the environmental or operational boundary conditions, no operational restrictions concerning the water depth and no risk of damaging the surroundings at the construction site by mooring or anchoring equipment. Furthermore, the very fast set-up at the location has to be mentioned, which leads to cost-efficient solutions even for short operations.

According to Holvik (1998) there are basically two different approaches for DP systems. On the one hand, there are DP systems that are only able to correct the deviation after it has happened. On the other hand, there are DP systems based on model control. These systems are able to start corrections before the deviation has actually occurred. This is possible because these systems are able to predict environmental forces and the resulting reaction of the vessel. Thus it is obvious that model controlled systems are more advanced but they need more input data as well.

A typical DP system consists of the power generation and management systems, a number of propulsion and maneuvering devices as well as the DP control systems. This is in accordance with IMCA (2007), where it is emphasized that a DP system includes the following three parts: power, control and references. Further, it is stated that power can be subdivided into power generation, distribution and consumption by the propulsion systems. Control is closer defined as the automatic or manual power management system as well as the position control system. Finally references are explained as essential sensors giving information about the position, the environment and the behavior of the vessel.

The importance of DP is growing because the number of offshore construction sites is increasing and thus more ships with DP capability are needed, e.g. for crane and jacking
operations. Since the DP capability of a vessel is directly connected to the hull form as well as the propulsion and maneuvering system, it is obvious that DP is a major design task. Hence, it should be taken into account during the early ship design phase in order to avoid the failure of a project or extensive man hours in order to secure the delivery of the ship. An example of the holistic consideration of DP during the early design stage of a new vessel can be found in Lübcke (2015b).

There are various designs of propulsion and maneuvering systems for vessels equipped with DP systems. In order to generate forces in the longitudinal and transverse direction as well as the yawing moment from zero to maximum loading, various combinations of propulsors with fixed directions, such as transverse tunnel thrusters as well as conventional main propellers, azimuth thrusters and combinations of these are feasible. The most efficient design of the propulsion and maneuvering system for each project has to be selected under consideration of the industrial mission of the ship. Large determining factors are the shares of dynamic positioning and transit during the operational life of the vessel in order to find the technically and economically best solution. If a vessel is basically always working in DP mode, the design will differ from a vessel which is working in transit mode most of the time and only smaller shares in DP mode.

The design of a DP system is very similar to the design of a slow speed maneuvering and propulsion system, as used for berthing of Ro-Ro vessels, for example. The determination of these systems can be split into two parts: On the one hand for the nominal forces each component involved has to deliver, a static approach can be used. This part can be solved based on equilibrium calculations for the longitudinal and transverse forces as well as for the yawing moment. On the other hand, a dynamic approach is needed. This is used to determine whether the system is capable of altering the required forces fast enough, in order to fulfill the DP task within the defined limits of motions of the vessel. For the dynamic approach time series of the external forces are required. Additionally, the machinery plant has to be considered, because the alteration of the forces depends on the reaction times of the engines. But the dynamic approach can not be solved successfully without a proper solution of the static approach.

2.2. Forces Acting on a Ship During Dynamic Positioning

The forces acting on a vessel during dynamic positioning can basically be subdivided into three parts:

- environmental forces
- forces from the propulsion and maneuvering system
• interaction forces

The environmental loads that have to be considered for dynamic positioning result from wind, sea current and waves. Wind forces result from the existent wind pressure multiplied by the relevant lateral area of the vessel and the dedicated wind drag coefficients. While wind forces can change fast in direction and strength, as it is known from sudden gusts, sea currents normally vary very slowly. This is due to the fact that sea currents are either tidal currents or result from wind coming from the same direction over a long period of time. The current forces acting on a ship with zero speed are basically the same as the resistance of a moving vessel. The third major share of the environmental loads is caused by the waves acting on the ship. At this point it has to be distinguished between first and second order wave forces. While first order wave forces are proportional to the wave amplitude in harmonic waves, they have a time-averaged mean value equal to zero. Thus these forces are not relevant for dynamic positioning. The wave forces that are relevant for dynamic positioning are the so-called wave drift forces. In Journée and Massie (2001) it is stated that for engineering purposes these are second order wave forces. Hence the wave drift forces are quadratic functions of the wave amplitude in harmonic waves and proportional to the square of the envelope of the wave excitation amplitude for irregular waves. Consequently, the time-averaged mean value is not equal to zero, resulting in slowly varying forces acting on the vessel. This makes the wave drift forces relevant for dynamic positioning.

This is also in accordance with Serraris (2009), where it is emphasized that only the low frequency motions originating from the wave drift forces, sea currents and wind loads should be considered for DP. High frequency motions, which would result from the first order wave forces, have to be filtered out in time domain DP computations. One reason for this is the response time of the thrusters of the DP system, which are not able to compensate for high frequency motions. Additionally, the whole propulsion system would suffer service life trying to compensate for the high frequency motions, which would result from the first order wave forces with a mean value of zero. Furthermore, the fuel consumption would most likely increase, too.

The differences between first and second order wave excitation forces are summarized in Fig. 10 according to Clauss et al. (1994).
The forces from the propulsion and maneuvering system of the ship are used in order to counteract the environmental forces. These forces are either the generated thrust of any kind of DP-device, e.g. main propellers and thrusters, or the hydrodynamic lift from a rudder. Finally, interaction forces have to be considered. The most important of this kind of forces are thruster-hull, thruster-thruster and thruster-current-interactions. A good example of thruster-hull-interactions is the Hovgaard-Effect, which represents the induced transverse force resulting from pressure and suction areas on the hull caused by the main propellers according to Sharma (1983). These forces can increase or decrease the effects of the propulsion and maneuvering system of a vessel and should therefore, if possible, already be considered in the early design process. If applicable, mooring and interaction forces between two vessels have to be taken into account in certain cases, e.g. if a pontoon is moored to a vessel in DP mode.

2.3. The Static Approach for Dynamic Positioning

The static approach for DP is used to determine the required force and power output of each involved component. This is done by equilibrium calculations of the forces acting in the longitudinal and the transverse direction of the vessel as well as the moment around the vertical axis. The static calculations are based on values of the environmental forces acting on the vessel, which are assumed to be constant in time. Consequently, the required propulsion forces are constant values as well.
2.3.1. The Elementary DP Equation System

\[ \begin{align*}
F_E & = F_{Ex} + F_{Ey} \\
N_E & = T_{PS} \cdot y_s - T_{STB} \cdot x_s
\end{align*} \]

Figure 11: Forces Acting on a Vessel in DP Mode with only the Main Propellers and the Bow Thruster(s) Involved

In the following, the DP task is solved with a propulsion system consisting of two main propellers and a number of bow thrusters, which are condensed to one resulting force. In this configuration the transverse force counteracting the environmental forces is produced solely by the bow thruster(s). This bow thruster force also leads to a yawing moment created by the force in combination with the longitudinal distance to the aft perpendicular of the vessel, which has to be compensated by the shoulder moment of the two main propellers. The shoulder moment is generated by the forwarding propeller on the windward side and the propeller in backing mode on the leeward side. Assuming that the environmental forces and moments are known, this arrangement leads to an equation system with three equations as well as three unknowns.

Figure 11 illustrates the relevant forces for a twin-screw vessel with bow thruster(s) for dynamic positioning. In the figure \( T_{PS} \) is the net thrust of the forwarding propeller on port side (windward side) and \( T_{STB} \) is the net thrust of the backing propeller on starboard side (leeward side). The distances of the main propellers from the center line are denoted \( y_s \). The nominal thrust of each propeller is decreased by the thrust deduction, which leads to the net thrust of each propeller at each specific thrust loading. \( F_B \) is the nominal force of the bow thruster(s). For the forces of lateral thrusters it is important to consider that these forces may drop significantly for non-zero forward speed as well as for currents or the combination of both. Deduction factors for the efficiency of transverse thrusters at forward speed may be found in Brix (1993) and the topic is also treated in Lübcke (2014). The time-averaged sum of the external forces acting on the vessel are denoted with \( F_E \) and the corresponding moment around the vertical axis is called \( N_E \).
For the following set up of the equation system a right handed coordinate system is
used with the x-axis pointing positive in forward longitudinal direction, the y-axis pointing
positive in port side transverse direction and the z-axis pointing positive upwards. The
origin of the coordinate system is located at the intersection point of the center line, the
base line and the aft perpendicular.

$$\Sigma F_x = 0 = T_{PS} - T_{STB} - F_{Ex}$$  \[(1)\]

In Eq. (1) the sum of all forces in longitudinal direction is supposed to be zero, in
order to hold the vessel in its position. $F_{Ex}$ is the sum of the external forces acting in
longitudinal direction. The equation shows that the forwarding propeller has to counteract
the longitudinal share of the external forces as well as the thrust of the backing propeller,
in case of bow quartering external forces as assumed in Fig. 11.

$$\Sigma F_y = 0 = F_B - F_{Ey}$$  \[(2)\]

In Eq. (2) the sum of all forces in transverse direction shall be zero in order to avoid a
transverse motion of the vessel. $F_{Ey}$ is the sum of the external forces acting in transverse
direction. This equation clearly shows that for this configuration only the bow thruster
is counteracting the transverse external forces.

$$\Sigma M_z = 0 = x_B \cdot F_B - N_E - y_s \cdot T_{STB} - y_s \cdot T_{PS}$$  \[(3)\]

Equation (3) shows the sum of all moments around the vertical axis, with $x_B$ being the
longitudinal position of the center of effort of the bow thruster force regarding to the aft
perpendicular. The sum of all moments is supposed to be zero in order to avoid a yaw
motion of the vessel.

The equation system consisting of Eqs. (1), (2) and (3) can be used to calculate the
required thrusts of the two main propellers as well as the bow thruster force, in order
to fulfill the DP task. It has to be emphasized that until here the DP equation system
consists of three equations and three unknowns. Thus the equation system is determinate
and the unknowns are the required transverse force of the bow thruster(s) as well as the
required thrusts of the windward and the leeward propeller.
2.3.2. Introducing the Windward Rudder into the DP Equation System

A properly designed full spade rudder located in the slipstream of a forwarding propeller is a very effective and cost-efficient generator of a transverse force due to its hydrodynamic lift. A twin-screw vessel will always be equipped with rudders and thus the rudders should be considered for the design of the DP system. At this point it is emphasized that only the rudder located in the slipstream of the forwarding propeller is effective and therefore only the windward rudder on port side according to Fig. 12 produces a transverse force. The rudder on the leeward side is not effective and thus neglected.

![Diagram of forces acting on a vessel in DP mode](image)

Figure 12: Forces Acting on a Vessel in DP Mode with the Main Propellers, Bow Thruster(s) and the Windward Rudder Involved

\[ \Sigma F_x \dagger = 0 = T_{PS} - T_{STB} - F_{E_x} - F_{R_x} \]  

(4)

In Eq. (4) the term \( F_{R_x} \) represents the longitudinal rudder force and it is obvious that this force is counteracting the thrust of the forwarding main propeller on port side. But the longitudinal rudder force is rather small, especially in comparison to the transverse force of the rudder.

\[ \Sigma F_y \dagger = 0 = F_B - F_{E_y} + F_{R_y} \]  

(5)

In Eq. (5) the transverse rudder force \( F_{R_y} \) is supporting the forces of the bow thruster(s) against the transverse share of the external forces.
$\Sigma M_z = 0 = x_B \cdot F_B - N_E - y_s \cdot T_{STB} - y_s \cdot T_{PS} + y_s \cdot F_{Rx}$ (6)

In Eq. (6) the point of load application of the longitudinal rudder force is the rudder stock on the windward side, which is in line with the center of the main propeller on port side. This distance combined with the longitudinal rudder force yields to a positive moment around the vertical axis, which therefore counteracts the shoulder moment.

Certainly the rudder forces depend on the angle of attack of the rudder and they are additional variables in the DP equation system. But by setting the rudder angle to a fixed value when the vessel is working in DP mode, the rudder forces solely depend on the thrust of the forwarding propeller. Consequently, it has to be emphasized that there are no additional unknown variables in the equation system, hence still having three equations for the three unknowns and thus the equation system is still determinate. The unknowns are still the required transverse force of the bow thruster(s) as well as the required thrusts of the windward and the leeward propeller.

2.3.3. Introducing Stern Thrusters into the DP Equation System

If the transverse force generated by the bow thruster(s) and the windward rudder is not large enough, the DP system can be extended by one or more stern thruster(s) as illustrated in Fig. 13. Since stern thrusters are lateral thrusters Eq. (4) stays the same, but Eq. (5) changes to Eq. (7) and Eq. (6) to Eq. (8), where $x_s$ is the longitudinal position of the center of effort of the stern thruster(s) referred to the aft perpendicular.
In combination with the stern thruster force this yields to a positive moment around the vertical axis in the right-handed system.

\[ \Sigma F_y \overset{!}{=} 0 = F_B - F_{E_y} + F_{R_y} + F_S \]  

\[ \Sigma M_z \overset{!}{=} 0 = x_B \cdot F_B - N_E - y_s \cdot T_{STB} - y_s \cdot T_{PS} + y_s \cdot F_{R_x} + x_s \cdot F_S \]  

The new equation system consisting of Eqs. (4), (7) and (8) contains more than three unknowns, while it still consists of three equations. As the stern thruster force does not depend on any of the other unknowns, the equation system is indeterminate and thus does not have a unique solution. Therefore, the solution of the equation system is an optimization problem now, for which several target functions may be formulated as described in Krüger and Vorhölter (2012).

Closing for this subsection, it has to be mentioned that the independent consideration of the forces is not correct, because some of the components involved in DP have strong interactions with each other. Especially the propeller-rudder and the propeller-hull interactions as well as the hull-thruster interactions in a current have to be accounted for.

2.4. Results of the Static DP Analysis

The results of a static DP analysis are force and moment equilibrium conditions for each encounter angle for the maximum environmental forces the vessel can counteract. These findings are typically presented in DP capability plots as shown in Fig. 14. These diagrams can be calculated for various environmental conditions and they show the capability of the vessel to maintain its position and heading dependent on the encounter angle and the wind speed.

The center of a DP capability plot represents the vessel and the angular lines represent the encounter angles, while each radial ring is valid for a certain wind speed. It is common practice to use Tab. 3 in order to choose corresponding significant wave heights and wave periods resulting from the wind speed under consideration. If applicable, the current velocity has to be selected individually. Wind, current and waves are normally taken as being coincident in direction as stated in Gonsholt and Nygard (2002). Since ship-shaped vessels commonly withstand higher environmental forces in longitudinal than in
transverse directions, capability plots mostly have larger expansion in vertical than in horizontal direction.

![Figure 14: Example of a DP Capability Plot](image)

Table 3: Relationship Between Significant Wave Height, Wave Period and Wind Speed Regarding IMCA (2000)

<table>
<thead>
<tr>
<th>Significant Wave Height $H_s$ [m]</th>
<th>Crossing Period $T_z$ [s]</th>
<th>Peak Period $T_p$ [s]</th>
<th>Mean Wind Speed $V_w$ [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>1.28</td>
<td>4.14</td>
<td>5.30</td>
<td>2.50</td>
</tr>
<tr>
<td>1.78</td>
<td>4.89</td>
<td>6.26</td>
<td>5.00</td>
</tr>
<tr>
<td>2.44</td>
<td>5.72</td>
<td>7.32</td>
<td>7.50</td>
</tr>
<tr>
<td>3.21</td>
<td>6.57</td>
<td>8.41</td>
<td>10.00</td>
</tr>
<tr>
<td>4.09</td>
<td>7.41</td>
<td>9.49</td>
<td>12.50</td>
</tr>
<tr>
<td>5.07</td>
<td>8.25</td>
<td>10.56</td>
<td>15.00</td>
</tr>
<tr>
<td>6.12</td>
<td>9.07</td>
<td>11.61</td>
<td>17.50</td>
</tr>
<tr>
<td>7.26</td>
<td>9.87</td>
<td>12.64</td>
<td>20.00</td>
</tr>
<tr>
<td>8.47</td>
<td>10.67</td>
<td>13.65</td>
<td>22.50</td>
</tr>
<tr>
<td>9.75</td>
<td>11.44</td>
<td>14.65</td>
<td>25.00</td>
</tr>
<tr>
<td>11.09</td>
<td>12.21</td>
<td>15.62</td>
<td>27.50</td>
</tr>
<tr>
<td>12.50</td>
<td>12.96</td>
<td>16.58</td>
<td>30.00</td>
</tr>
<tr>
<td>13.97</td>
<td>13.70</td>
<td>17.53</td>
<td>32.50</td>
</tr>
<tr>
<td>15.49</td>
<td>14.42</td>
<td>18.46</td>
<td>35.00</td>
</tr>
</tbody>
</table>
While the typical DP class notation regarding redundancy and reliability of the DP system is defined by the International Maritime Organization and the classification societies, the DP capability has to be specified by the owner in accordance with the industrial mission of the vessel. However, according to DNV (2012) the DP capability has to be demonstrated to the classification society anyway and the relevant documentation becomes part of the operating manual of the vessel upon approval. This is done in the way of DP capability plots. According to DNV (2011) DP capability plots should be available for different environmental conditions in accordance to the operational sites of the vessel. In addition, there should be envelope curves at each environmental condition for at least the following three operational situations:

1. fully intact power generation and all thrusters available
2. loss of most effective thruster(s)
3. following the worst case failure

As emphasized in von Ubisch (2004) it has to be mentioned that the DP capability plots are depending on the calculation methodologies used. Thus the results can vary for the same ship calculated with different software. Consequently, it is always advisable not only to look at the DP capability plot but also to ask for the methodologies used during the computations.

In addition to the documentation of the DP capability of an existing vessel, a static DP analysis can be used to show differences of DP configurations during the design process. The wider the envelope curve, the better the station keeping capability of the ship design. This is especially important when it is not possible to freely choose the heading of the vessel for a DP operation. One reason for this are the routes of underwater cables in wind farms. In this case the vessel has to be positioned in a way that it does not destroy the cables during the jacking operation. Furthermore it is obvious that the hull form as well as the superstructure have strong effects on the DP behavior of a vessel. Therefore, the DP capability plots have to be recalculated if large changes of the superstructure or the hull of a vessel are carried out. This is true for the ship design stage as well as for conversions, e.g. if a different crane is installed. Especially during the early design of a new ship a large number of different combinations and configurations regarding the DP system have to be evaluated with computations. In order to fulfill this goal a fast and reliable code is required.

Even though dynamic effects are sometimes considered in DP capability plots based on a static analysis by using safety factors, the two major problems about the static DP
analysis are the use of constant wind and wave drift forces as well as the negligence of the machinery plant. By neglecting these dynamic factors the static analysis lacks the insight into the heading and position accuracy of the DP system. Especially if mean values are used for the constant environmental forces, the DP system will not be able to compensate for the higher external forces in situations where all DP components are working at their maximum power limitations. This has to be pointed out, because the time-dependent forces exceed their mean values for about 50% of the time. Furthermore, the capability of the vessel to react to changes of the environmental forces strongly depends on the ability of the machinery plant to generate the required power in a sufficient period of time. Therefore, time domain assessment is needed for the evaluation of the dynamic behavior of a vessel working in dynamic positioning mode. This can be done either by expensive model tests or by time domain computations, as stated in van’t Veer and Gachet (2011) as well.

2.5. The Dynamic Approach for Dynamic Positioning

The computation of dynamic positioning in time domain can basically be considered as a maneuvering problem. Therefore, the existing force model for maneuvering according to Söding (1984) and Krüger (1998) can be used. Figure 15 illustrates a basic explanation of this model as it is shown in Haack (2006), where the details about the engaged propulsion system can also be found.

Figure 15: Layout of the Maneuvering Model

The focus of maneuvering computations is on the changed track due to the variation of
the rudder angle as well as the resulting increase of the resistance of the vessel, which has to be overcome by higher loads of the machinery and propulsion plant. The determination of the present position of the vessel in each time step is also part of the state vector for maneuvering problems. For the DP problem the deviation of the present position and heading of the vessel compared to the target values has to be determined additionally in each time step. The enhancement of the maneuvering force model for the solution of the time domain DP problem is described in Lübcke (2015a) and summarized in the following.

For each angle of attack selected for the static DP analysis a time domain computation is carried out. The static results of the equilibrium condition of one encounter angle are used for the initialization of the model. This includes the required thrust determined during the static analysis for each component involved. Thus the required combinations of pitch and revolutions can be found from the open-water propeller characteristics for each component. Hence the required torques are known and consequently the needed positions of the fuel racks are set as well.

Based on the calculated maximum values for the environmental forces in the static analysis, time series for the wind and wave drift forces are needed. The current forces are assumed to be constant over the computation time. The approach for the time-dependent wind forces can be found in Lübcke (2012) and the generation of time signals for wave drift forces will be explained in detail later. With the initialized model and the time series of the environmental forces the DP time domain computation can be carried out.

In each time step the environmental forces are acting on the vessel and consequently the vessel may change the position and the heading if the forces are strong enough. Based on this, the difference regarding the target position and heading is calculated and compared to the threshold value. If the deviation of the vessel caused by the environmental forces is beneath the threshold value, the controller is not acting in order to avoid excessive sensitivity of the system. If a threshold value is exceeded, the controller activates the propulsion control plant in order to counteract the environmental forces.

The entity of the results of the time domain computation for each encounter angle is the DP footprint of the vessel under consideration of the dynamics of the machinery plant and the time-dependent environmental forces. The DP footprint therefore contains the deviations of the position and heading of the vessel for the selected environmental forces. If the deviations of the position and/or the heading are higher than allowed, this can be caused either by not enough maximum thrust of the engaged propulsion components, an insufficient dynamic of the machinery plant or the combination of both. Consequently,
the permissible environmental forces have to be reduced until the vessel is able to keep
the deviations small enough, or the DP system has to be improved in order to fulfill the
specified DP capability.

2.6. Conclusions

It is obvious that a dynamic DP approach is necessary and for its success two elementary
entities have to be settled: On the one hand, the results of the static analysis have to
be meaningful and, on the other hand, there must be a functional relation between the
alteration rate of the maneuvering forces and the DP problem. If these two constraints
come true, the dynamic functionality of the whole DP system solely depends on the
maximum possible alteration rates of the forces of each component involved in the dynamic
positioning task. This in turn is directly related to the capability of the main engines
to get along with the required alteration rates of the power demand of all maneuvering
devices.
3. Frequency Domain Computation of the Horizontal Wave Drift Forces and the Wave Drift Yaw Moment

3.1. Introduction and State of the Art

For the design of dynamic positioning systems it is necessary to compute the exciting horizontal wave drift forces and the wave drift yaw moment in the early design stage. Basically there are three options for the determination of these forces and the yaw moment: wave basin tests, the use of a database and numerical computations. Wave basin test are rather expensive and, in order to use a database, data are required, which is not really applicable for the design of new ship types. Therefore, numerical computations are a very important way for the determination of wave drift forces in the early design stage. As mentioned above, the use of such direct calculation methods increases the possibility of success for a project.

The numerical determination of mean wave drift forces is well known in literature either by far field or near field formulations. The basis for the far field method was introduced by Maruo (1960) and extended by Newman (1967) for the yaw moment. The near field method was introduced by Pinkster and van Oortmerssen (1977). The idea of the far field formulation is the conservation of momentum, while the near field formulation is based on direct pressure integration over the wetted hull surface. Both methodologies have been further developed by various authors throughout the world and should lead to the same results as shown in Hung and Taylor (1988). However, this cannot always be ensured in practice as pointed out in Papanikolaou et al. (2011).

In Söding (2014a) three second-order potential flow methods for the computation of wave drift forces and the wave drift yaw moment are discussed. The three methods are a three-dimensional Rankine-source method, a strip method and an unconventional new method based on Lee (1982). It is accentuated that viscosity does not influence wave-induced forces a lot, thus the use of potential flow methods is applicable. The results of all three methods are compared to experimental data and to the results of a RANS method for two different ship types. Even though the focus of the article is upon the added resistance of ships in waves, it is emphasized that the findings are valid for the horizontal wave drift forces and the wave drift yaw moment as well. This is due to the fact that the wave-induced additional resistance for vessels with forward speed is basically the same as the negative longitudinal wave drift force as mentioned in Papanikolaou and
Zaraphonitis (1987). The basic findings in Söding (2014a) are that the potential flow and RANS computations as well as model experiments of wave drift forces and the wave drift yaw moment in general are not very accurate, yet. This is especially true for waves shorter than half the length between perpendiculars of a vessel. Still, it is testified that the Rankine-source method and the unconventional new method are roughly applicable for the use in ship design, but it is pointed out that for this fine meshes are needed, especially when the waves are short. The strip method is only recommended for the determination of drift forces, when the waves are longer than eighty percent of the length between perpendiculars of the ship. Furthermore, it is highlighted that the RANS computations, at least in longer waves, appear comparable to model test and thus may be useful for the validation of the much faster potential flow methods. Finally, it is mentioned that RANS computations are still too slow for wave drift force predictions in irregular waves.

Anyhow, there is an extremely strong need for a fast and rather easy-to-use method for the prediction of the wave drift forces and the wave drift yaw moment already within the early ship design. For this reason the methodology described as strip theory above is further examined in the following. This methodology uses the first order motions of the vessel and the first order pressure on the ship's hull in the waterline for the determination of the second order wave drift forces and yaw moment in the frequency domain. Additionally, the influence of the quadratic velocity term in Bernoulli's equation is surveyed. In order to overcome the aforementioned problems regarding short waves, the methodology is supplemented by a correction for short waves. According to Papanikolaou and Liu (2016) the focus on the correct short wave prediction is increasing lately, because of the continuous increase in ship sizes. Consequently, the ratio between wave length and ship length is shifted towards smaller values and thus the share of relatively short waves is increasing.

The necessity for the short wave correction is the complete reflection of short waves on the hull, leading to a finite asymptotic value for the wave drift forces and the yaw moment. However, the wave drift forces and the wave drift yaw moment become zero for long waves, because the waves are transmitted beneath the vessel, leading to negligible diffraction effects as illustrated in Fig. 16. Between these two asymptotic cases the waves are partly reflected and partly transmitted. Furthermore, the drift forces and the yaw moment have peaks corresponding with resonance effects in the heave, roll and pitch motions between the two asymptotic boundaries as stated in DNV (2014).
3.2. Theory of the Frequency Domain Approach

In 1970 Boese published his theory for the calculation of the added resistance in waves in the frequency domain (Boese, 1970). A short description of the approach from Boese can also be found in Söding (1982). This theory is the basis for the following explanations on how to substitute the pressure integration over the ship’s hull by the use of the first order motions of the vessel. For this two basic components are identified, which are responsible for the time-averaged longitudinal forces:

1. the forces on the hull up to the static waterline (mean waterline)
2. the forces on the immersing and emerging part of the hull in waves

In the following, two right-handed coordinate systems are used: the ship-fixed system \((x, y, z)\) and the inertial system \((\xi, \eta, \zeta)\). Both systems have the origin at the intersection point of the midship section, the center line and the base line of the considered vessel. Figure 17 shows the inertial coordinate system used for the description of the theory.

3.2.1. The Longitudinal Wave Drift Force on the Hull up to the Static Waterline

In the ship-fixed coordinate system the longitudinal force on the hull up to the static waterline \(F_x\) can be found by pressure integration according to Eq. (9). The corresponding vertical force \(F_z\) is shown in Eq. (10).

\[
F_x = -\int_{S_0} p \cos(\delta) \, dS 
\]

\[
F_z = -\int_{S_0} p \cos(\kappa) \, dS 
\]
In the equations $S_0$ stands for the mean water surface, $p$ represents the water pressure on the hull, $\delta$ is the angle between the normal vector on the hull surface and the ship-fixed longitudinal $x$-axis and $\kappa$ is the same regarding the vertical $z$-axis. The operator $dS$ represents a surface element of the hull. At this point the use of the ship-fixed system is useful, because the normal vectors on the hull surface are time independent despite the periodic motion of the vessel. Using the linearized pressure, the forces in the ship-fixed system are harmonic values.

The target of the following methodology is the substitution of the pressure integration. For this it is reasonable to have a closer look at the longitudinal force in the inertial system $F_\xi$. Starting with the consideration of the pitch motion, $F_\xi$ can be written as Eq. (11) with the pitch angle $\vartheta$.

$$F_\xi = F_x \cos(\vartheta) + F_z \sin(\vartheta)$$ \hspace{1cm} (11)

Linearization of the pitch motion leads to Eq. (12).

$$F_\xi \approx F_x + F_z \cdot \vartheta$$ \hspace{1cm} (12)

By assuming small motions the vertical force can be expressed by Newton’s second law
of motion. With the total mass of the vessel \( m \) and the vertical acceleration of the mass center of gravity of the vessel \( \dot{\zeta}_s \), this leads to Eq. (13).

\[
F_z \approx F_\zeta = m \cdot \ddot{\zeta}_s
\] (13)

Entering Eq. (13) into Eq. (12) the time-averaged mean value of the longitudinal forces \( F_\xi^2 \) can be written by way of Eq. (14). The overbar symbolizes time-averaged values and the exponent implies the second order. The first order harmonic value \( F_x \) has a time-averaged mean value equal to zero and thus it is disregarded. The product of the first order harmonic values \( \ddot{\zeta}_s \) and \( \dot{\vartheta} \) is of second order and thus the time-averaged mean value has to be considered. In the following * denotes conjugate-complex values.

\[
\overline{F_\xi^2} = m \cdot \overline{\ddot{\zeta}_s \cdot \dot{\vartheta}}
\]

\[
= m \cdot \text{Re} \left( -\omega_e^2 \ddot{\zeta}_s e^{i\omega_e t} \right) \cdot \text{Re} \left( \dot{\vartheta} e^{i\omega_e t} \right)
\]

\[
= m \cdot \text{Re} \left( -\omega_e^2 \ddot{\zeta}_s e^{i\omega_e t} \frac{1}{2} \left[ \dot{\vartheta} e^{i\omega_e t} + \dot{\vartheta}^* e^{-i\omega_e t} \right] \right)
\]

\[
= m \cdot \left[ \text{Re} \left( -\omega_e^2 \ddot{\zeta}_s \frac{1}{2} \dot{\vartheta} e^{2i\omega_e t} \right) \right] + m \cdot \text{Re} \left( -\omega_e^2 \ddot{\zeta}_s \frac{1}{2} \dot{\vartheta}^* \right)
\]

The complex amplitude of the heave motion of the mass center of gravity of the vessel is \( \ddot{\zeta}_s \). The corresponding complex transfer function \( \hat{Y}_{3a} \) is defined as the ratio of the complex amplitude of this motion and the complex amplitude of the wave causing that motion. This leads to Eq. (15):

\[
\ddot{\zeta}_s = \hat{Y}_{3a} \cdot \ddot{\zeta}_a
\] (15)

Further \( \hat{Y}_{5*} \) is the complex transfer function of the conjugate-complex pitch motion \( \dot{\vartheta}^* \) of the vessel and \( |\ddot{\zeta}_a|^2 \) is the square of the absolute value of the exciting complex wave amplitude.

Substituting the motions of the vessel by the complex transfer functions and neglecting the part with the zero mean value, Eq. (14) becomes Eq. (16). This is the equation for the resulting longitudinal wave drift force on the hull up to the static waterline. Until now only the heave and pitch motions have been considered.
\[ F_{\xi}^2 = -\frac{m \cdot \omega_r^2}{2} \text{Re} \left( \hat{Y}_s \cdot \hat{Y}_s^* \right) \cdot |\hat{\omega}|^2 \]  

(16)

### 3.2.2. The Longitudinal Wave Drift Force on the Immersing and Emerging Part of the Hull in Waves

The calculation explained in the previous subsection only takes into account the pressure fluctuation over the hull up to the static waterline. Hence it is obvious that there needs to be a correction for the longitudinal wave drift force on the immersing and emerging part of the hull in waves. The derivation of the corresponding equation starts with Fig. 18. On the left side the wave crest situation and on the right side the wave trough situation is shown.

![Figure 18: Pressure Alteration due to Relative Motion and Resulting Horizontal Force](image)

In Fig. 18 \( z_R \) denotes the relative motion between the vessel and the incoming wave, while \( F_H \) is the horizontal force per waterline element, resulting from the water pressure on the hull. \( F_H \) is defined positive when pointing inward into the vessel. \( h \) is the vertical height from the static waterline and \( p(h) \) is the pressure at this distance from the mean waterline.

Equation (17) shows the dependency of the the time-dependent pressure \( p(h,t) \) on the time-dependent relative motion, where \( \rho \) is the density of the water and \( g \) is the gravitational acceleration.

\[ p(h,t) = \rho g (h - z_R(t)) \]  

(17)
According to Eq. (18) the horizontal force per waterline element is positive for \( z_R < 0 \). This positive force has to be added to the force obtained up to the static waterline as shown in the previous subsection.

\[
F_H = \int_{0}^{-z_R} p(h, t) \, dh = \frac{1}{2} \rho g z_R^2
\]  

(18)

For \( z_R > 0 \) the horizontal force is negative as shown in Eq. (19).

\[
F_H = \int_{0}^{z_R} p(h, t) \, dh = -\frac{1}{2} \rho g z_R^2
\]  

(19)

This negative force has to be subtracted from the force obtained up to the static waterline as shown in the previous subsection. Thus for the wave crest as well as for the wave trough condition the force up to the static waterline has to be corrected by addition of Eq. (20) for each waterline element.

\[
F_H = \frac{1}{2} \rho g z_R^2
\]  

(20)

Based on the horizontal force per waterline element \( F_H \) the resulting longitudinal force on the vessel \( F_\xi \) can be determined. In Fig. 19 the relation between these two forces is illustrated. The angle of the waterline element versus the longitudinal axis of the vessel is denoted by \( \alpha \).

![Figure 19: Relation of \( F_H \) and \( F_\xi \)](image)

In order to obtain the resulting longitudinal force from the horizontal forces on all waterline elements, the integration along the waterline is needed. This leads to Eq. (21), where \( dl \) stands for a waterline line element.
3 FREQUENCY DOMAIN COMPUTATION OF THE HORIZONTAL WAVE DRIFT FORCES AND THE WAVE DRIFT YAW MOMENT

\[
F_\xi = -\oint_{WL} \frac{1}{2} \rho g z_R^2 \sin \alpha \, dl
\]  \hspace{1cm} (21)

With Eq. (22) the length of the waterline elements can be substituted, which leads to Eq. (23).

\[
dx = dl \cdot \cos \alpha
\]  \hspace{1cm} (22)

\[
F_\xi = -\oint \frac{1}{2} \rho g z_R^2 \tan \alpha \, dx
\]  \hspace{1cm} (23)

The closed curve integral can be substituted by the sum of the integration along each side of the vessel. Furthermore, Eq. (24) is valid and the complex transfer function of the relative motion is \( \hat{Y}_{zR} \). Thus Eq. (25) is true. Combining this with Eq. (14) leads to the equation for the time-averaged longitudinal wave drift force on the immersing and emerging part of the hull in waves \( \overline{F^2_\xi} \) as shown in Eq. (26).

\[
\tan \alpha = -\frac{dy}{dx}
\]  \hspace{1cm} (24)

\[
\hat{Y}_{zR} \cdot \hat{Y}_{zR}^* = |\hat{Y}_{zR}|^2
\]  \hspace{1cm} (25)

\[
\overline{F^2_\xi} = \frac{1}{4} \rho g \sum_{Stb,Port} \int_L |\hat{Y}_{zR}|^2 \frac{dy^+_w}{dx} \, dx \cdot |\hat{\xi}_a|^2
\]  \hspace{1cm} (26)

In the equation above \( y_W \) is the y-coordinate of the waterline. Furthermore, \( y^+_W = y_W \) for the starboard side of the vessel and \( y^-_W = -y_W \) for port side. So normally \( y^+_W = |y_W| \). However, there are two possible cases, when \( y^+_W \) becomes negative: on the one hand if \( y_W > 0 \) on port side and on the other hand if \( y_W < 0 \) on starboard side.
3.2.3. The Resulting Equation for the Longitudinal Wave Drift Force

Equation (27) shows the formulation of the longitudinal wave drift force as it is published in Söding and Bertram (2013).

\[
\overline{F_2} = \frac{1}{2} \left( m \omega_e^2 \text{Re} \left( \hat{Y}_2 s \hat{Y}_6 - \hat{Y}_3 s \hat{Y}_5 \right) \right) + \frac{1}{2} \rho g \sum_{Stb, Port} \int_L |\hat{Y}_{zt}|^2 \frac{dy^+}{dx} \, dx + \frac{1}{2} \rho g \left( |\hat{Y}_4|^2 + |\hat{Y}_5|^2 \right) \cdot \left[ A_{x0} \left( T + z_{x0} \right) \right]_{\text{Transom}} \right) \zeta_2^2
\]

The first term of Eq. (27) accounts for the longitudinal force on the hull of the vessel beneath the static waterline. This term is basically the same as Eq. (16), but it is extended for the impact of the sway and the yaw motions of the vessel. This can be done analogue to the impact of the heave and pitch motions as described above. The second term considers the force on the ship due to the relative motion between the vessel and the water surface. This term is identical with Eq. (26). The third term is a time-averaged correction term valid for ships with an unsubmerged transom beneath the waterline at forward speed. Therefore, the third term is irrelevant for the zero speed case during dynamic positioning. This term describes the instantaneous pressure on the transom area of the ship due to the motions of the ship in waves, even though the transom is dry in the stationary flow field, hence taking into account the resulting positive longitudinal force on the vessel. In this term \( z_{x0} \) is the vertical coordinate of the center of area of the vessel in its static floating position and \( A_{x0} \) is the corresponding sectional area. Furthermore, \( T \) denotes the submergence of the origin under the still water surface and, finally, \( \hat{Y}_4 \) is the complex transfer function of the roll and \( \hat{Y}_5 \) of the pitch motion.

3.2.4. Equations for the Transverse Wave Drift Force and the Wave Drift Yaw Moment

In the same manner as shown above for the longitudinal wave drift force, the following equations for the time-averaged transverse wave drift force as well as for the time-averaged wave drift yaw moment can be derived. Equation (28) is valid for the horizontal transverse wave drift force and may also be found in Söding and Bertram (2013), while Eq. (29) is true for the wave drift yaw moment and may be found in an unpublished paper by Söding.
3 FREQUENCY DOMAIN COMPUTATION OF THE HORIZONTAL WAVE DRIFT FORCES AND THE WAVE DRIFT YAW MOMENT

\[
\overline{F_{\eta}}^2 = \frac{1}{2} \left( - \omega_e^2 \Re \left( \hat{Y}_{1s} \hat{Y}_6^* \right) \right) + \rho g \sum_{Stb,Port} \int_L y_w^+ \Re \left( \hat{Y}_{zd0} \hat{Y}_{\zeta W_y}^* \right) \, dx \right) \zeta_a^2
\]

\[
\overline{M_{\zeta}}^2 = -\frac{1}{2} \omega_e^2 \Re \left[ \hat{\alpha}^* \times \Theta \hat{\alpha} \right]_\zeta + \sum_{WL-Elements} \left( (\vec{x} - \vec{x}_G) \times \left[ \frac{|p_w|^2}{4 \rho g} \Delta_s \times (0, 0, -1)^T \right] \right) \zeta + x_g \overline{F_{\eta}}^2 - y_g \overline{F_{\xi}}^2
\]

In Eq. (29) the index \( \zeta \) indicates the relevant vector component for the yaw moment. The last two terms are required because the origin of the coordinate system is not the mass center of gravity of the vessel.

3.3. Solution of the Equations for the Horizontal Wave Drift Forces and the Wave Drift Yaw Moment

For the solution of the equations for the horizontal wave drift forces and the wave drift yaw moment, the complex transfer functions for the relevant degrees of freedom and the complex transfer function of the relative vertical motion between the vessel and the waves are needed. These first order results may be calculated with a variety of different sea-keeping codes.

For this thesis the required first order motions of the vessel are calculated according to the strip method as described in Söding (1969). This is a robust code, which leads to short computation times and it is known for reliable results for cargo and navy vessels. The required hydrodynamic coefficients are calculated based on potential flow theory, thus neglecting viscous effects.

In order to consider radiation, diffraction and Froude-Krilov effects during the determination of the relative motion between the water surface and the ship, the complex transfer function of the pressure at the static waterline can be used. The relation between the complex transfer functions of the relative vertical motion \( \hat{Y}_{zd} \) and the pressure \( \hat{Y}_p \) is given in Eq. (30).
The first order pressures on the hull surface are determined with the derivation of the pressure strip method by Söding and Blume (1994), which is based on Hachmann (1991). While usually the calculation of pressures is omitted in common strip methods, this is the focus of the latter method.

A critical examination concerning the applicability of the strip theory is necessary, because of the unusual main dimension ratios of wind farm installation vessels and the fact that strip methods are based on slender body assumptions.

However, the length/beam ratio of the SIETAS TYPE 187 is 3.67, still significantly larger than the lower threshold value of 2.5, for which a remarkable effectiveness for the prediction of ship’s motion by strip theory is testified in Faltinsen (1990). In addition the use of the strip theory is approved with good results for wave length as short as one third of the vessel’s length in Bertram (2000), which leads to critical wave frequencies higher than approximately $1 \, \text{s}^{-1}$ for the vessel under consideration. Hence if the discretization of the strips is done properly, the large beam/draft ratio of the vessel should not be a problem. However, this is to be validated in the present thesis.

### 3.4. Description of the Computation Model

The computation model of the SIETAS TYPE 187, as provided from the shipyard, consists of the hull form, light ship distribution, room arrangement and a couple of loading conditions. All of these are actual data from the early design stage, when the wave basin tests are conducted. Hence there are differences to the final design.

Figure 20 shows the computation model as used for the calculation of the complex transfer functions using strip theory.
The red cuboid in the figure above has the equivalent mass properties as the vessel in the examined loading condition. The wave basin tests regarding the wave drift forces are conducted in two loading conditions. Both loading conditions are on the design draft. The difference between the loading conditions is the position of the legs. In the first condition the legs are in the upward and in the second loading condition in the downward position. The results for the wave drift forces and the wave yaw moment only differ very slightly between the two loading conditions, even though the vertical mass center of gravity and the underwater lateral area change quite significantly. The very similar results for both loading conditions are most likely due to the fact that the wave drift forces and the wave drift yaw moment act for the most part in or in a narrow band around the static waterline. Since the static waterline between the two loadings conditions does not differ, the results should be rather similar.

Thus for the computations for this thesis the first loading condition with the legs in the upward position is chosen. In this loading condition the vessel is floating with even
keel on its design draft. A more precise description of the loading condition is beyond the scope of the agreement on publishing data of the vessel.

The following figures show the motion response amplitude operators (MRAOs) of the SIETAS TYPE 187 in the computed loading condition. The MRAOs are the absolute values of the complex transfer functions of the corresponding motion. Each diagram shows the MRAOs computed for the same sets of encounter angles and wave frequencies. All calculations are done considering the unit wave amplitude.

Figure 21(a) is valid for the surge motion of the vessel and Fig. 21(b) shows the MRAOs for the sway motions.

Since the MRAOs are not compared to measured data, only general statements are possible. It can be stated that the principal shape of the curves above is meaningful. This can be said because, for example, the values converge against zero for short waves. Furthermore, the surge MRAOs for following and head waves converge against one for long waves, while the same is true for the sway MRAO in long beam waves.
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Figure 22: MRAOs for Heave and Roll of SIETAS TYPE 187

Figure 22(a) illustrates the MRAOs for the heave motion. For very long waves the vessel follows the wave amplitude with its heave motions. This is correct and can be found in the figure. Fig. 22(b) shows the MRAOs for the roll motion of the vessel. Here the distinct roll resonance frequency can be found and the roll MRAOs converge against one for long waves. This is reasonable because the vertical axis of the diagram is nondimensional with the wave slope.

In Fig. 23(a) the MRAOs for the pitch and in Fig. 23(b) for the yaw motion are shown. In following and head waves the MRAOs for the pitch motion have to converge against one in this diagram, as found.

Figure 23: MRAOs for Pitch and Yaw of SIETAS TYPE 187

Special focus has to be put on Figures 21(b), 22(b) and 23(b). At the first glance it is not reasonable that the vessel sways, rolls and yaws in either following or head seas in regular waves. The answer to this is the very asymmetric light ship weight distribution...
of the vessel regarding the center line. This is supported by the shape of the cuboid in Fig. 20. A big contribution to this is the weight of the huge crane, which is located on port side aft, with its foundation around the aft leg. This leads to a large absolute value for the roll-pitch-coupling term $\Theta_{xy}$, which is the moment of deviation regarding the longitudinal and transverse axes of the vessel. This value is even increased by the required ballast water to balance the trim and heel of the vessel, especially if it is located in a tank diagonally across the vessel thus on the starboard side at the front of the vessel. The large roll-pitch-coupling term is also the leading reason why, for example, the MRAOs of the roll motion for encounter angles of $135^\circ$ and $225^\circ$ are not identical. Next to $\Theta_{xy}$ the pitch-yaw-coupling term $\Theta_{yz}$ as well as the non-zero transverse coordinate of the mass center of gravity of the vessel influence the MRAOs, too. However, their influence on this vessel in this loading condition is negligible compared to $\Theta_{xy}$. To illustrate this $\Theta_{xy}$ is set to zero and the resulting MRAOS are presented in the following figures.

Comparing Fig. 24(a) with Fig. 21(a), Fig. 25(a) with Fig. 22(a) and Fig. 26(a) with Fig. 23(a) it can be stated that there are no significant changes. However, when comparing Fig. 24(b) with Fig. 21(b), Fig. 25(b) with Fig. 22(b) and Fig. 26(b) with Fig. 23(b) the influence of $\Theta_{xy}$ becomes clear.

![MRAOs Surge](figure24a.png)

![MRAOs Sway](figure24b.png)

Figure 24: MRAOs for Surge and Sway of SIETAS TYPE 187 with $\Theta_{xy} = 0$
In this subsection the results for the aforementioned way to compute wave drift forces are presented. These results are valid for the SIETAS TYPE 187. Each of the following diagrams shows the horizontal wave drift forces or the wave drift yaw moment computed for the same sets of encounter angles and wave frequencies, as used for the determination of the MRAOS. All calculations are also done with unit wave amplitude. The results are presented in the following coordinate system: the system is right handed, the longitudinal axis is positive in the ship's forward direction, the transverse axis is positive to port side and the vertical axis is positive in the upward direction. Consequently, the longitudinal forces are positive forward, the transverse forces are positive in port side direction and the yaw moment is positive turning counterclockwise around the vertical axis. The encounter angles are also counted counterclockwise from $\mu = 0^\circ$ for following seas, over the starboard side to $\mu = 180^\circ$ for head seas and from there back over the port side of the
vessel.

Figure 27 shows the results for the longitudinal wave drift forces and Fig. 28 for the transverse wave drift forces.

![Figure 27: Frequency Domain Results for $T_{\xi}^2$](image1)

![Figure 28: Frequency Domain Results for $T_{\eta}^2$](image2)

In Fig. 29 the results for the wave drift yaw moment are displayed. All three diagrams
are based on the set of complex transfer functions corresponding to the MRAOs presented in Figs. 21(a) to 23(b).

![Graph showing wave drift moment of yaw](image)

Figure 29: Frequency Domain Results for $M_\zeta^2$

During the introduction of this chapter it was mentioned that the wave drift forces and the wave drift moment have asymptotic boundaries for long and short waves. The asymptotic boundary for long waves, which is equal to zero, can be found in Figs. 27, 28 and 29. However, for short waves there are no asymptotic values computed. This will be dealt with later in this thesis.

In comparison, the wave drift forces and the wave drift yaw moment based on the set of MRAOs computed for $\Theta_{xy} = 0$ are presented in the following. Thus the following three diagrams are based on the set of complex transfer functions corresponding to the MRAOs presented in Figs. 24(a) to 26(b).

Figure 30 shows the results for the longitudinal wave drift forces, Figure 31 for the transverse wave drift force and in Fig. 32 the results for the wave drift yaw moment are displayed.
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Figure 30: Frequency Domain Results for $F_{\xi}^{2}$ with $\Theta_{xy} = 0$

Figure 31: Frequency Domain Results for $F_{\eta}^{2}$ with $\Theta_{xy} = 0$
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Figure 32: Frequency Domain Results for $\bar{M}_\zeta$ with $\Theta_{xy} = 0$

The asymptotic boundary for long waves, which is equal to zero, can also be found in Figs. 30, 31 and 32. However, for short waves there are again no asymptotic values.

Comparing Figs. 27 and 30 it can be stated that they differ around the roll resonance frequency of the vessel. This had to be expected because that is where the MRAOs also differ. The rest of the two diagrams is very similar to each other. Looking at the transverse wave drift forces, namely Figs. 28 and 31, it can be noticed that they do not differ at all. Finally Figs. 29 and 32 also differ around the roll resonance frequency. However, they also differ in the area of shorter waves, with the wave drift yaw moments for $\Theta_{xy} = 0$ being more symmetrical for both sides of the vessel.

From this point on the results for $\Theta_{xy} = 0$ will not be further investigated, since the focus is upon the real loading condition.

3.6. Frequency Domain Calculations under Consideration of the Quadratic Velocity Term

Until now the pressure has been linearized, which means that the quadratic velocity term in Bernoulli’s equation is omitted as shown in Eq. (31). In this equation, the first term is the dynamic and the second term is the hydrostatic pressure. The dynamic pressure is of small magnitude and has been omitted as well.
3 FREQUENCY DOMAIN COMPUTATION OF THE HORIZONTAL WAVE DRIFT FORCES AND THE WAVE DRIFT YAW MOMENT

\[
p = -\rho \frac{\partial \Phi}{\partial t} - \rho gh \tag{31}
\]

Milgram (2003) emphasized that the added resistance in waves of full shaped vessels is over-predicted if the quadratic velocity term is neglected. Therefore, its inclusion is recommended, even though other terms of comparable magnitude are still omitted. This is also in accordance with Söding (2014b), where the fluctuation of the water velocity along the shell is emphasized to be considered for the determination of the wave drift forces and the wave drift moment of yaw. The reason for this is the second order contribution of the velocity to the pressure, as found in the unsteady formulation of Bernoulli’s equation as shown in Eq. (32). This directly lead to the formulation for the pressure as shown in Eq. (33).

\[
\frac{\rho}{2} |\vec{v}|^2 + p + \rho gh + \rho \frac{\partial \Phi}{\partial t} = \text{constant} \tag{32}
\]

\[
p = -\rho \frac{\partial \Phi}{\partial t} - \rho gh - \frac{\rho}{2} |\vec{v}|^2 \tag{33}
\]

The absolute value of the velocity vector can be expressed in dependency on the complex velocity potential in way of Eq. (34).

\[
|\vec{v}| = \sqrt{\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2} \tag{34}
\]

\[
\text{Re} \left(a e^{i\omega t}\right) \cdot \text{Re} \left(b e^{i\omega t}\right) = \frac{1}{2} \text{Re} \left(ab^*\right) \tag{35}
\]

Considering Eq. (35) for the time-averaged mean value of a complex number and applying it to Eq. (34), the time-averaged mean value for the complex pressure is found. The first term of Eq. (33) representing the dynamic pressure is of small magnitude and thus may be neglected. The second term is already considered in the determination of the wave drift forces and the wave drift yaw moment above. Thus only the third term has to be considered additionally. With the last two equations this term results in Eq. (36):
According to Söding and Bertram (2014) the numerical solution is carried out as an extension to the strip method used above. The hull of the vessel between two strip offset sections is approximated by triangles. The corners of the triangles are pressure points. With the periodic potential known at each corner, the average gradient of the potential for each triangle can be calculated. The gradient of the potential is the same as the velocity vector $\vec{v}$ as shown in Eq. (34). Thus the additional pressure term for each triangle is known according to Eq. (36). Thereafter, this pressure term has to be multiplied with the respective area vector of each triangle to find the additional shares of the wave drift forces and the wave drift yaw moment.

The results for the longitudinal wave drift force considering the quadratic velocity term are shown in Fig. 33, for the transverse force in Fig. 34 and for the wave drift yaw moment in Fig. 35.

Figure 33: Frequency Domain Results for $F_\xi^2$ Considering the Quadratic Velocity Term
Comparing Fig. 33 with Fig. 27, it can be stated that the consideration of the quadratic velocity term makes the results for following seas more reasonable, because they no longer change the algebraic sign for shorter waves. Looking at the transverse wave drift force by comparing Fig. 34 with Fig. 28, it is obvious that the absolute values for the transverse forces drop significantly for shorter waves. The comparison of Fig. 35 with Fig. 29 shows that the consideration of the quadratic velocity term leads to more symmetrical wave drift.
yaw moments for both sides of the vessel.

The asymptotic boundary for long waves, which should be zero, can still be found in Figs. 33, 34 and 35. However, for short waves there are still no asymptotic values and hence a correction for short waves is introduced in the next subsection.

3.7. Frequency Domain Calculations with Corrections for Short Waves and under Consideration of the Quadratic Velocity Term

Faltinsen (1990) pointed out that wave drift forces for long waves originate from the motions of the vessel, while drift forces in short waves are mainly due to the reflection of the incoming waves from the ship's hull. For the longer waves the aforementioned equations can be used, whereas the calculation for the zero speed case for short waves is done in accordance to Faltinsen (1990), as explained below. Figure 36 shows the definitions used for the following equations.

![Figure 36: Definitions for the Short Wave Correction based on (Faltinsen, 1990).](image)

Equation (37) shows how the longitudinal wave drift forces are calculated in short waves, while Eq. (38) is valid for the transverse wave drift forces and Eq. (39) for the wave drift yaw moment.

\[
F_{\xi \text{ short}}^2 = -\frac{\rho g \zeta^2}{2} \int_{L_1} \sin^2 (\gamma + (\pi - \mu)) \cdot \sin (\gamma) \, dl \tag{37}
\]
\[ F_{\eta_{\text{short}}}^2 = -\frac{\rho g \zeta_a^2}{2} \int_{L_1} \sin^2 (\gamma + (\pi - \mu)) \cdot \cos(\gamma) \, dl \] (38)

\[ M_{\zeta_{\text{short}}}^2 = -\frac{\rho g \zeta_a^2}{2} \int_{L_1} \sin^2 (\gamma + (\pi - \mu)) \cdot (\xi_0 \cdot \cos(\gamma) + \eta_0 \cdot \sin(\gamma)) \, dl \] (39)

The basic physical assumption for this approach is a standing wave. This standing wave is caused by the superposition of two waves with the same wave length and amplitude but opposite directions. The two waves are an incoming regular wave and its total reflection at the ship’s side, which is idealized as a vertical wall. If the vessel is not or only slightly moving because the waves are short, there are no waves generated by the vessel through radiation and if the incoming wave is totally reflected, there is no transmission beneath the vessel. Additionally, no dissipation, for example, through friction losses on the ship’s hull are considered. The amplitude of the reflected wave in this case is identical with the amplitude of the incoming wave and it can never be larger. Therefore, the maximum wave elevation of the standing wave at the ship’s vertical side can not be larger than twice the incident wave amplitude. This is sketched in Fig. 37.

Figure 37: Superposition of Incoming and Totally Reflected Wave at a Vertical Wall Results in a Standing Wave with Twice the Incident Wave Amplitude

The integration of the hydrostatic pressure results in the maximum time-averaged force as shown in Eq. (40).
\[ F_{\text{maxshort}}^2 = -\frac{\rho g \zeta^2}{2} \]  

This is the first term of Eqs. (37), (38) and (39) and also the reason for the asymptotic values of the wave drift forces and yaw moment as shown in the following figures. The integral in the equations above takes into account varying angles between the wave encounter direction and the orientation of the waterline element.

At this point the question about when waves are short has to be discussed. In Faltinsen (1990) wavelengths are small regarding the added resistance in waves, when they are not longer than half the length of the ship. Söding (2014b) says that for waves shorter than about eighty percent of the length between perpendiculars the strip method is unsuitable for the determination of the added resistance in waves. The reason for this is that the basic assumption for the strip theory may no longer apply. This basic assumption is the two-dimensional periodic circulation of the water around the ship in frame sections, without having substantial longitudinal components.

![Figure 38: Frequency Domain Results for \( F_{\xi}^2 \) Considering the Quadratic Velocity Term including Short Wave Correction](image)

Figure 38 shows characteristic results for the longitudinal wave drift force including the short wave correction applied for waves shorter than the length between perpendiculars of the vessel. For the SIETAS TYPE 187 this corresponds to \( \omega = 0.68 \text{ s}^{-1} \). The boundary
line for the short wave correction is set to this position in order to get a physically useful shape of the curves representing the longitudinal wave drift forces.

Figure 39: Frequency Domain Results for $F^2$ Considering the Quadratic Velocity Term including Short Wave Correction

In Fig. 39 the results for the transverse drift force including the short wave correction applied for waves shorter than the length of the vessel are shown.

Figure 40 shows the results for the wave drift yaw moment, where the short wave correction is also applied for waves shorter than the length between perpendiculars of the vessel. Again the boundary line for the short wave correction is set to this position in order to get physically useful shapes of the curves representing the transverse wave drift forces as well as the wave drift yaw moment.
In all three diagrams the graphs show finite asymptotic values for short waves, vanishing values for long waves and in between peaks corresponding with resonance effects of the ship’s motion.

It has to be mentioned that the idealization of the ship’s side as a vertical wall is certainly not true for all waterline elements, especially at the bow of the vessel.

3.8. Frequency Domain Calculations with Corrections for Short Waves but without Consideration of the Quadratic Velocity Term

For the sake of completeness the following three diagrams show the results for the frequency domain calculations with the corrections for short waves but without the consideration of the quadratic velocity term.

Figure 41 shows the results for the longitudinal wave drift forces, Fig. 42 for the transverse wave drift forces and Fig. 43 for the wave drift moments of yaw.
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Figure 41: Frequency Domain Results for $F_\xi^2$ with Short Wave Correction

Figure 42: Frequency Domain Results for $F_\eta^2$ with Short Wave Correction
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In summary the frequency domain calculations of the wave drift forces as well as the wave drift yaw moment as described in this chapter can be used for the static layout of components used for dynamic positioning. However, for the holistic evaluation of a complete DP system time domain computations are required. This is already emphasized in Chapter 2. For the time domain computations time series of the horizontal wave drift forces and the wave drift yaw moment are needed. These time series can be generated on basis of the frequency domain results for the unit wave amplitude, as explained in the next chapter. When having time series available, it is easy to obtain corresponding mean values for the static layout of the DP components. Consequently there is no need to use the frequency domain results directly for the layout of the DP components.

Figure 43: Frequency Domain Results for $\overline{M}_{\zeta}^2$ with Short Wave Correction

3.9. Conclusions
4. Time Domain Computation of the Horizontal Wave Drift Forces and the Wave Drift Yaw Moment

4.1. Introduction

In the following section the results from the frequency domain approach previously described are used as input data for the computation of time series for the horizontal wave drift forces and the wave drift yaw moment. The frequency domain results are based on the unit wave amplitude and sets of encounter angles and wave frequencies.

The description of the presented time domain computation procedure for the horizontal wave drift forces and the wave drift yaw moment in long-crested irregular waves can also be found in Clauss et al. (1994). Based on this, an improvement for a better approximation of the natural seaway by short-crested irregular waves is then introduced.

4.2. Theory of the Time Domain Approach for Long-Crested Irregular Waves

The theory described in this section has been developed for the evaluation of flexible mooring systems for offshore applications, just like conventional chain and/or cable anchoring of floating offshore structures as well as mooring to articulated towers or others. As one can understand, the consideration of the random characteristics of the horizontal wave drift forces and the wave drift yaw moment are important for the evaluation of such flexible mooring systems.

As mentioned at the beginning of this thesis, station-keeping can either be achieved by fixed mooring or dynamic positioning systems. Even though these two station-keeping approaches are very different, the wind loads, sea currents and wave drift forces the systems have to withstand are the same. This is the reason why it seems reasonable to use this theory for the determination of time series for the horizontal wave drift forces and the wave drift yaw moment for dynamic positioning.

The following theory is based on the classic superposition model for the probabilistic description of the stationary seaway. The basic assumption for the superposition model is the ability to describe the wave elevation at any point of the water surface as a random superposition of elementary waves. These elementary waves are assumed to be regular and have different lengths, heights and propagation directions. Furthermore, the waves are
assumed to be linear, thus satisfying the conditions of the linear (Airy) wave theory. The random character of the seaway is generated by uniformly distributed phase angles. Seaway data from energy density spectra is used in order to generate realistic natural seaways.

The determination of the time-dependent forces is done with Eq. (41). Depending on the selected input data it is valid for the calculation of time series for the longitudinal and transverse wave drift forces as well as the wave drift yaw moment.

\[ F^2(t) = \frac{\rho}{2} \cdot g \cdot L \cdot \alpha^2_0 \cdot a^2(t) \]  

Equation (41) shows the dependency of the wave drift forces on the time-dependent square of the envelope \( a^2(t) \) of the random seaway. This envelope of the random seaway corresponds to the amplitude fluctuation of the random seaway under consideration. Furthermore, the dependency on the mean drift force coefficient \( \alpha^2_0 \) becomes clear.

The mean drift force coefficient is defined by Eq. (42). This equation clearly shows that the mean drift force coefficient is a weighted average to the seaway spectrum.

\[ \alpha^2_0 = \frac{\int_0^\infty \alpha^2(\omega) \cdot S_{zz}(\omega) \, d\omega}{\int_0^\infty S_{zz}(\omega) \, d\omega} \]  

Equation (43) represents the definition of the drift force coefficient \( \alpha^2(\omega) \). Depending on the input data this equation is valid either for the longitudinal wave drift forces, the transverse wave drift forces or the wave drift yaw moment. The equation shows how the drift force coefficient is determined from the frequency domain results \( F^2(\omega) \) with \( \zeta_a^2 \) being the square of the unit wave amplitude.

\[ \alpha^2(\omega) = \frac{F^2(\omega)}{\frac{\rho}{2} \cdot g \cdot L \cdot \zeta_a^2} \]  

At this point only the time signal for the square of the envelope of the random seaway remains for the solution of Eq. (41). At the beginning of this section it was mentioned that the envelope of the random seaway corresponds to the amplitude fluctuation of the random seaway. So basically the square of the envelope is equal to the square of the absolute value.
of the complex wave amplitude $|\hat{\zeta}|^2$ and this can be expressed as in Eq. (44).

$$|\hat{\zeta}|^2 = \left[ \sqrt{\left(\text{Re}(\hat{\zeta})\right)^2 + \left(\text{Im}(\hat{\zeta})\right)^2} \right]^2$$  \hspace{1cm} (44)

Equations (45) shows the dependency of the real part of the complex wave amplitude and the time-dependent wave amplitude. Thus the first term in Eq. (44) can be substituted.

$$\text{Re}(\hat{\zeta}) = \zeta(t)$$  \hspace{1cm} (45)

Now a substitution for the second term needs to be found. This can be done by the time derivation of $\zeta(t)$ according to Eq. (46).

$$\frac{d\zeta}{dt} = \text{Re}(i\omega_0 \hat{\zeta})$$  \hspace{1cm} (46)

$$= \text{Re} \left( i\omega_0 [\zeta_r + i\zeta_i] \right)$$
$$= -\omega_0 \zeta_i$$
$$= -\omega_0 \text{Im}(\hat{\zeta})$$

In the equation above $\zeta_r$ is the real and $\zeta_i$ the imaginary part of the complex wave amplitude. Equation (46) leads to the expression for the imaginary part of the complex wave amplitude as stated in Eq. (47).

$$\text{Im}(\hat{\zeta}) = -\frac{d\zeta}{dt} \frac{1}{\omega_0}$$  \hspace{1cm} (47)

The insertion of Eqs. (45) and (47) into Eq. (44) leads to Eq. (48).

$$|\hat{\zeta}|^2 = \zeta^2(t) + \left(\frac{d\zeta}{dt}\right)^2 \frac{1}{\omega_0^2}$$  \hspace{1cm} (48)

Equation (48) can also be expressed in terms of Eq. (49). This is the equation for the square of the envelope presented in Clauss et al. (1994).
\[ a^2(t) = \zeta^2(t) + \frac{(d\zeta/dt)^2}{\omega_0^2} \] (49)

Based on Eq. (50) the time series for the square of the envelope of the random seaway can be determined with Eq. (49) and thus time series for the horizontal wave drift forces and the wave drift yaw moment can be computed. In the following equation the wave spectrum is subdivided into the number of J parts.

\[ \zeta(t) = \sum_{j=1}^{J} \sqrt{2 \cdot S_{zz}(\omega_j) \Delta \omega_j} \cdot \text{Re} \left( e^{i(\omega_j t + \epsilon_j)} \right) \] (50)

### 4.3. Results of the Time Domain Approach for Long-Crested Irregular Waves

The results for the aforementioned time domain computation procedure are presented below. These diagrams show time series of $10^4$ seconds. This is the recommended order of magnitude emphasized in Clauss et al. (1994). Furthermore, it is pointed out that the statistical evaluation has to be done over the whole length of this time series in order to achieve reasonable results for the purpose of design decisions, such as changes in dimensions or alterations of a configuration. The length of each time step is 0.5 seconds. The long-crested irregular seaway is generated by superposition of 48 wave frequencies with randomly chosen phase angles. Furthermore, the seaway is based on a Pierson-Moskowitz spectrum with a significant wave height of $H_{1/3} = 1.8 \text{ m}$, a peak period of $T_p = 7.5 \text{ s}$ and for the encounter angle is $\mu_0 = 135^\circ$.

Figure 44 shows the time series for the longitudinal wave drift force of the SIETAS TYPE 187. This diagram is based on the frequency domain results presented in Fig. 38. The mean value of the full time series of $10^4$ seconds for the longitudinal wave drift force is 25.90 kN.
Figure 44: Time series of $F^2_\xi(t)$ for SIETAS TYPE 187 in Long-Crested Irregular Waves based on a Pierson-Moskowitz Spectrum with $H_{1/3} = 1.8 \text{ m}$, $T_p = 7.5 \text{ s}$ and $\mu_0 = 135^\circ$.

The results for the transverse wave drift force of the SIETAS TYPE 187 are shown in Fig. 45 and they are based on the frequency domain results presented in Fig. 39. The mean value of the full time series of $10^4$ seconds for the transverse wave drift force is 89.68 kN.

Figure 45: Time series of $F^2_\eta(t)$ for SIETAS TYPE 187 in Long-Crested Irregular Waves based on a Pierson-Moskowitz Spectrum with $H_{1/3} = 1.8 \text{ m}$, $T_p = 7.5 \text{ s}$ and $\mu_0 = 135^\circ$.

Figure 46 shows the results for the wave drift yaw moment of the SIETAS TYPE 187.
based on the frequency domain results presented in Fig. 40. Finally, the mean value of the full time series of $10^4$ seconds for the wave drift yaw moment is 3735.86 $kNm$.

![Graph showing wave drift moment of yaw over time](image)

Figure 46: Time series of $M_\zeta^2(t)$ for SIETAS TYPE 187 in Long-Crested Irregular Waves based on a Pierson-Moskowitz Spectrum with $H_{1/3} = 1.8 m$, $T_p = 7.5 s$ and $\mu_0 = 135^\circ$.

### 4.4. Improvement of the Time Domain Approach for Short-Crested Irregular Waves

Adapted from the computation procedure for long-crested irregular waves described above, this subsection deals with the calculation of time series for wave drift forces and the wave drift yaw moment in short-crested irregular waves. This is done by the introduction of an angular spreading function. Applying Eq. (51), as stated in Perez (2005), and combining Eq. (51) with Eq. (42), this leads to Eq. (52).

$$\int_0^\infty S_{zz}(\omega) \, d\omega \approx \sum_{j=1}^J \frac{\zeta_j^2(\omega_j)}{2}$$  \hspace{1cm} (51)

$$\alpha_0^2 = \frac{\sum_{j=1}^J \alpha_j^2(\omega_j) \cdot \zeta_j^2(\omega_j)}{\sum_{j=1}^J \zeta_j^2(\omega_j)}$$  \hspace{1cm} (52)

As described in Söding (1982) the complex wave amplitude for a short-crested seaway
can be determined with Eq. (53) by using Eq. (54):

\[ \hat{\zeta}(\omega_j, \mu_l) = \sqrt{2S_{zz}(\omega_j, \mu_l) \Delta \omega_j \Delta \mu_l} e^{i \epsilon_{j,l}} \]  

(53)

where:

\[ S_{zz}(\omega, \mu) = \frac{1}{2} \cos^2 (\mu - \mu_0), \]  

(54)

for \( |\mu - \mu_0| \leq \frac{\pi}{2} \), else \( S_{zz}(\omega, \mu) = 0 \). 

In Eq. (55) the relation between the real and the complex wave amplitude is shown. The real wave amplitude is equal to the absolute value of the complex wave amplitude.

\[ \zeta(\omega_j, \mu_l) = |\hat{\zeta}(\omega_j, \mu_l)| \]  

(55)

Based on Eq. (52) and using Eq. (55) with Eq. (53) and Eq. (54), the mean drift force coefficient for short-crested irregular waves is derived according to Eq. (56) as follows:

\[ \alpha_0^2 = \frac{\sum_{j=1}^{J} \sum_{l=1}^{L} \alpha^2(\omega_j, \mu_l) \cdot \zeta^2(\omega_j, \mu_l)}{\sum_{j=1}^{J} \sum_{l=1}^{L} \zeta^2(\omega_j, \mu_l)} \]  

(56)

Based on the frequency domain results the wave drift force coefficients can be determined according to Eq. (57).

\[ \alpha^2(\omega, \mu) = r_{\omega}^2(\omega, \mu) \cdot \frac{g}{L} \cdot \frac{\zeta_a}{\zeta_a^2} \]  

(57)

In order to generate the required time signal of the wave amplitude, Eq. (58) from Söding (1982) can be adopted as follows:

\[ \zeta(t) = \sum_{j=1}^{J} \sum_{l=1}^{L} \sqrt{2S_{zz}(\omega_j, \mu_l) \Delta \omega_j \Delta \mu_l} \cdot \Re \left( e^{i(\omega_j t + \epsilon_{j,l})} \right) \]  

(58)

Finally, using Eq. (56), Eq. (57) and Eq. (58) instead of Eq. (42), Eq. (43) and Eq. (50) for the solution of Eq. (41), the procedure described above for the time domain compu-
tation of the horizontal wave drift forces and the wave drift yaw moment for long-crested irregular waves can now be used with the better approximation of the natural seaway by short-crested irregular waves as well.

4.5. Results of the Time Domain Approach for Short-Crested Irregular Waves

The results for the aforementioned time domain computation procedure for the horizontal wave drift forces and the wave drift yaw moment in short-crested irregular waves are presented below. Again the diagrams show time series of $10^4$ seconds simulation time with time steps of 0.5 seconds. The short-crested irregular seaway is also generated by superposition of 48 randomly chosen wave components: 6 encounter angles and 8 wave frequencies. Furthermore, the seaway is based on a Pierson-Moskowitz spectrum with a significant wave height of $H_{1/3} = 1.8 \text{ m}$, a peak period of $T_p = 7.5 \text{ s}$ and for the encounter angle is $\mu_0 = 135^\circ$.

Figure 47 shows the time series for the longitudinal wave drift force of the SIETAS TYPE 187 in short-crested waves. This diagram is based on the frequency domain results presented in Fig. 38. The mean value of the full time series of $10^4$ seconds for the longitudinal wave drift force is $25.97 \text{ kN}$.

![Figure 47: Time series of $F_2^\xi(t)$ for SIETAS TYPE 187 in Short-Crested Irregular Waves based on a Pierson-Moskowitz Spectrum with $H_{1/3} = 1.8 \text{ m}$, $T_p = 7.5 \text{ s}$ and $\mu_0 = 135^\circ$](image)

The results for the transverse wave drift force are shown in Fig. 48 of the SIETAS
TYPE 187 in short-crested waves. This diagram is based on the frequency domain results presented in Fig. 39. The mean value of the full time series of $10^4$ seconds for the transverse wave drift force is $88.43 \, kN$.

Figure 48: Time series of $F_{\eta}^2(t)$ for SIETAS TYPE 187 in Short-Crested Irregular Waves based on a Pierson-Moskowitz Spectrum with $H_{\frac{1}{3}} = 1.8 \, m$, $T_p = 7.5 \, s$ and $\mu_0 = 135^\circ$.

Figure 49: Time series of $M_{\zeta}^2(t)$ for SIETAS TYPE 187 in Short-Crested Irregular Waves based on a Pierson-Moskowitz Spectrum with $H_{\frac{1}{3}} = 1.8 \, m$, $T_p = 7.5 \, s$ and $\mu_0 = 135^\circ$.

Figure 49 shows the results for the wave drift yaw moment of the SIETAS TYPE 187 in short-crested waves. This diagram is based on the frequency domain results presented.
in Fig. 40. Finally, the mean value of the full time series of $10^4$ seconds for the wave drift yaw moment is $3665.75 \, kNm$.

### 4.6. Conclusions

In this section it is shown how the frequency domain results can be used for the determination of time series for the horizontal wave drift forces and the wave drift yaw moment. This is done for both long-crested irregular waves and short-crested seaways.

The comparison of Figs. 44 and 47 for the longitudinal wave drift forces, Figs. 45 and 48 for the transverse wave drift forces and Figs. 46 and 49 for the wave drift moment of yaw illustrates that the whole time series differ a lot from each other for the long- and short-crested seaway. The corresponding mean values of the complete time series are nearly the same for each of the forces and the yaw moment. This leads to the fact that, on the one hand, for the static dynamic positioning approach the differences between the different kinds of seaway play a minor role. However, on the other hand, it clearly points out the importance of the dynamic approach for dynamic positioning. This is due to the fact that the dynamic forces between the two different seaways lead to completely different scenarios the DP system of the vessel has to deal with.

The presented computational procedure for the determination of the horizontal wave drift forces and the wave drift yaw moment can also be used to generate input data for time domain simulations of dynamic positioning maneuvers based on impulse response functions as presented in Detlefsen et al. (2015).
5. Validation of the Computational Procedure

5.1. Introduction

Validation is the assurance that a product fulfills the requirements it is supposed to. In this case, the results of the computational procedure explained above will be compared to model test results. By doing this, it has to be proven that the strip theory, which is based on slender body assumptions, leads to reasonable results for ships with blunt, flat and wide hull forms. Furthermore, it has to be demonstrated that the time domain approach, originally developed for flexible mooring systems of offshore structures, can be used for the determination of time series of the horizontal wave drift forces as well as the wave drift yaw moment used for dynamic positioning computations.

For the validation of the presented calculation procedure wave basin test results for the SIETAS TYPE 187 in unidirectional irregular waves are compared to computed results.

5.2. Test Set-up and Program

The wave basin tests have been carried out at FORCE TECHNOLOGY in Denmark. The layout of the test set-up is roughly sketched in Fig. 50.

![Figure 50: Rough Sketch of the Test Set-up in Following Seas (Top View)](image)

The model in scale 1:27 is fixed by four mooring lines as shown above. The upstream mooring lines are equipped with linear springs. Each mooring line is connected to a xy-gauge. These gauges measure the forces in longitudinal and transverse tank directions. Since it is only possible to generate waves in the longitudinal direction of the wave-basin,
the model has to be rotated in order to enable different encounter angles. This is realized by adjusting the mooring lines and thus the heading of the model in the tank. Since the forces are measured in tank coordinates, they have to be transformed into the coordinate system of the vessel afterwards. The longitudinal and transverse wave drift forces acting on the model are calculated by the summation of the corresponding forces measured at each of the four xy-gauges. No measurements for the wave drift yaw moment have been carried out.

The measurement of the longitudinal and transverse wave drift forces have been performed for the three sea states listed in Tab. 4. For each of the sea states encounter angles of 180°, 225°, 270°, 315° and 360° have been realized. Based on the model scale each run of the measurements is equal to 3.600 s in full scale.

<table>
<thead>
<tr>
<th>Name</th>
<th>Spectrum</th>
<th>$H_{1/3}$ [m]</th>
<th>$T_p$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>W4</td>
<td>Pierson-Moskowitz</td>
<td>1.8</td>
<td>7.5</td>
</tr>
<tr>
<td>W5</td>
<td>Pierson-Moskowitz</td>
<td>1.8</td>
<td>11.3</td>
</tr>
<tr>
<td>W6</td>
<td>Pierson-Moskowitz</td>
<td>1.8</td>
<td>15.0</td>
</tr>
</tbody>
</table>

The irregular seaway for the computation is composed of forty-eight wave components with random phase angles. Since the sea-keeping tests have been performed with unidirectional seaways no angular spreading is used, thus leading to a long-crested seaway. The size of each time step is 0.5 s.

The comparison of the measured values and the computed results for the longitudinal and transverse wave drift forces is carried out by the mean values of the time series, because only these measurements were available. No measurements for the wave drift yaw moment have been carried out and thus no validation is possible.

For the computation the modified Pierson-Moskowitz spectrum according to Eq. (59) from Söding (1982) is used:

$$S_{zz}(\omega) = 173 \frac{H_{1/3}^2}{T_1^4} \omega^{-5} e^{-692T_1^4\omega^{-4}}$$

(59)

The relation between the peak period and the average period for a Pierson-Moskowitz spectrum can be found in Fossen (2011) and it is defined according to Eq. (60).

$$T_1 = 0.771 \cdot T_p$$

(60)
5.3. Comparison of Measured and Calculated Data without the Quadratic Velocity Term and without Short Wave Correction

In this section the data measured for the sea states specified in Tab. 4 are compared to the results of the calculations without the quadratic velocity term and without short wave correction. This is done for computation times of 3.600 s and 10.000 s. The comparisons of the results are presented in the following six diagrams. The computations are based on the frequency domain results presented in Figs. 27 and 28.

![Diagram](a) Computation Time: 3,600 s

![Diagram](b) Computation Time: 10,000s

Figure 51: Comparison of the Mean Values of the Wave Drift Forces for Sea State W4 Calculated without the Quadratic Velocity Term and without Short Wave Correction

In Fig. 51(a) the results for a computation time of 3.600 s and in Fig. 51(b) for 10.000 s are compared to the measured data for the sea state W4. It can be stated that the differences between the calculated mean values from the time series computed for 3.600 s and 10.000 s are negligible for this sea state.

Furthermore, it is obvious that the computed results for the longitudinal wave drift force in following and head seas are too small. This had to be expected when looking at the frequency domain results in Fig. 27, because these results for short waves are most likely too small, as already explained in Chapter 3. The computed results for the transverse wave drift force are not really convincing, since the computed values for bow and stern quartering waves are over-predicted, while the results for beam seas are too small.

In Figs. 52(a) and 52(b) the computed results for the two computational times are compared to the measured data for sea state W5. Again the differences between the calculated mean values for the two time periods are insignificant.
Since W5 has a larger peak period than W4 the share of longer wave components increase. The agreement of the computed results for the longitudinal wave drift force with the measurements is rather good for following waves, while they are too high for head seas. The computed results for the transverse wave drift forces are higher than the measured values for stern and bow quartering waves as well as for beam seas.

When comparing Figs. 53(a) and 53(b) it is obvious that the differences between the calculated mean values for 3,600 s and 10,000 s can be neglected for W6 as well.

The comparison of the calculated to the measured data can be summarized as follows: Except for the longitudinal wave drift forces in head seas, which are over-predicted, the
computed results match the measured data rather well for W6. For the transverse wave drift forces the agreement for head, following and beam seas is very good, while the forces for bow and stern quartering waves are over-predicted by the calculations.

5.4. Comparison of Measured and Calculated Data with the Quadratic Velocity Term but without Short Wave Correction

This section shows the comparison of the measured data to the results computed under consideration of the quadratic velocity term. The short wave correction is still not applied. Contrary to the last section the measurements are only compared to computed results for time periods of 3,600 s. The reason for this is the negligible difference between the mean values for the two computation time periods as shown above.

The considered sea states are again the ones listed in Tab. 4. The computed results shown in the following three diagrams are based on the frequency domain results presented in Figs. 33 and 34.

Figure 54: Comparison of the Mean Values of the Wave Drift Forces for Sea State W4 Calculated with the Quadratic Velocity Term but without Short Wave Correction for a Computation Time of 3600 s

Figure 54 shows the comparison for sea state W4. Due to the introduction of the quadratic velocity term the values for the transverse wave drift forces for stern and bow quartering waves as well as for beam seas drop significantly. The results for the longitudinal wave drift forces also change due to the introduction of the quadratic velocity term, but the values still differ from the measured data.
Figure 55: Comparison of the Mean Values of the Wave Drift Forces for Sea State W5 Calculated with the Quadratic Velocity Term but without Short Wave Correction for a Computation Time of 3600 s

Figure 55 shows the comparison of the computed and measured data for W5. The measured and computed mean values agree very well for the longitudinal and transverse forces with two exceptions. Firstly, the computed longitudinal wave drift force in stern quartering waves is too large. Secondly, the computed transverse wave drift force in beam seas is smaller than the measurement.

Figure 56: Comparison of the Mean Values of the Wave Drift Forces for Sea State W6 Calculated with the Quadratic Velocity Term but without Short Wave Correction for a Computation Time of 3600 s

Figure 56 compares the data for W6. Basically the results match very well. However,
there are three exceptions: For the longitudinal wave drift force in stern quartering and following waves the computed results are too high and the computed transverse wave drift force in beam seas is too small.

5.5. Comparison of Measured and Calculated Data with the Quadratic Velocity Term and with Short Wave Correction

In the following the measured data are compared to the computed results under consideration of the quadratic velocity term as well as the short wave correction. Again the sea states from Tab. 4 and the computed results for time periods of 3.600 s are used. The calculated longitudinal wave drift forces shown in the following are based on the frequency domain results presented in Fig. 38. However, for the transverse wave drift force the frequency domain results presented in Fig. 57 instead of Fig. 40 are used. This is due to the fact that during the validation the mean values based on the time series of the transverse wave drift forces with short wave correction for waves shorter than half the length between perpendicular was found to fit better to the measured data.

![Figure 57: Frequency Domain Results for $F^2_\eta$ Considering the Quadratic Velocity Term and with Short Wave Correction for Waves Shorter than 0.5 Lbp](image)

Figure 58 illustrates the comparison of the mean values of the longitudinal and transverse wave drift forces for sea state W4. The longitudinal forces for head and following waves match rather well, but for stern quartering waves they are slightly over and for bow quartering waves under-predicted. The computed results for the transverse wave drift forces are too small in comparison to the measured data.
Figure 58: Comparison of the Mean Values of the Wave Drift Forces for Sea State W4 Calculated with the Quadratic Velocity Term and with Short Wave Correction for a Computation Time of 3600 s

Figure 59 is valid for W5. The longitudinal wave drift forces match rather well, except for head and following waves. In these two cases the computed values are too high in comparison to the measured data. The transverse wave drift forces fit very well. Only the computed value of the transverse wave drift force in beam seas is smaller than the measurement.

Figure 59: Comparison of the Mean Values of the Wave Drift Forces for Sea State W5 Calculated with the Quadratic Velocity Term and with Short Wave Correction for a Computation Time of 3600 s

For Fig. 60 and thus for sea state W6, the statement about the agreement of the
calculated and measured forces is the same as for Fig. 59.

Figure 60: Comparison of the Mean Values of the Wave Drift Forces for Sea State W6 Calculated with the Quadratic Velocity Term and with Short Wave Correction for a Computation Time of 3600 s

5.6. Comparison of Measured and Calculated Data without the Quadratic Velocity Term but with Short Wave Correction

The computed results under consideration of the short wave correction but without the quadratic velocity term are compared to the test results in this section. Again the sea states from Tab. 4 are used and the computation period is 3.600 s.

Figure 61: Frequency Domain Results for $\overline{F_\theta^2}$ without the Quadratic Velocity Term but with Short Wave Correction for Waves Shorter than 0.5 Lbp
The data shown in the following are based on the frequency domain results presented in Fig. 41 for the longitudinal wave drift forces. However, for the transverse wave drift force the results shown in Fig. 61 instead of Fig. 42 are used. As in the section before, this is due to the fact that during the validation the transverse wave drift forces with short wave correction for waves shorter than half the length between perpendicular was found to fit better to the measured data.

Figure 62 is valid for sea state W4. The mean values of the longitudinal wave drift forces match well for head, following and beam seas. For bow quartering waves the computed mean values are smaller and for stern quartering waves larger than the measured values. The mean values of the computed transverse wave drift forces also fit rather well. Only the computed mean value for beam seas is smaller than the measured value. However, still it fits better to the measured data than the values presented above for the same sea state. The calculated mean value for stern quartering waves is slightly larger compared to the measurement.

![Figure 62: Comparison of the Mean Values of the Wave Drift Forces for Sea State W4 Calculated without the Quadratic Velocity Term but with Short Wave Correction for a Computation Time of 3600 s](image.png)

In Fig. 63 the computed and measured results for sea state W5 are compared. The mean values for the longitudinal wave drift forces for following and head seas are over-predicted by the calculations, while the values for stern and bow quartering as well as beam seas are in good accordance with the test results. The mean values of the transverse wave drift forces for bow and stern quartering as well as beam seas are over-predicted by the calculations in comparison to the test results.
Figure 63: Comparison of the Mean Values of the Wave Drift Forces for Sea State W5 Calculated without the Quadratic Velocity Term but with Short Wave Correction for a Computation Time of 3600 s

Figure 64 compares the measured and computed results for sea state W6. The mean values of the longitudinal wave drift forces match well, except for head and following waves. Here the calculated values are too high in comparison to the measured data. The transverse wave drift forces for bow and stern quartering waves are over-predicted by the calculations when compared to the test results, but the other values match very well.

Figure 64: Comparison of the Mean Values of the Wave Drift Forces for Sea State W6 Calculated without the Quadratic Velocity Term but with Short Wave Correction for a Computation Time of 3600 s
5.7. Conclusions

The highest mean values of the horizontal wave drift forces are measured for sea state W4. This is the sea state with the shortest peak period and therefore with the largest share of short waves. For the design of a DP system the focus concerning the wave drift forces and the wave drift moment of yaw should thus be on the short waves in order to avoid an undersized DP system. Hence it is more important to predict the short waves accurately and rather accept an over-prediction of the longer waves than the opposite.

Following from the validation it can be stated that the strip method is well-suited for the determination of wave drift forces in longer waves. The wave drift forces in short waves are higher than predicted by the strip theory. The short wave correction presented above also works well. However, the critical question arises: When are waves short and thus should be treated with the short wave corrections? This question is not easily answered.

As always when comparing computed to measured data there are some uncertainties where the deviations come from. In this case there might, for example, be differences between the target and achieved values of the sea states or problems by modeling the weight distribution and, therefore, the moments of inertia and deviation during the wave basin tests. On the other hand, the described theory on which the presented calculation procedure is based, has inaccuracies as well. However, pointing out that the wave drift forces are second order forces, which are computed based on first order results here, and assuming an accuracy e.g. of $10 - 15\%$ for the first order results, the accuracy of the second order results may not be better than $20 - 30\%$ as it is mentioned in Salvesen (1978). This leads to the fact that the presented results are reasonable for all three sea states.

From a ship design point of view rather conservative results are favorable for the dimensioning of a DP-Systems. Thus the results without the consideration of the quadratic velocity term but with short wave correction as presented in Figs. 62, 63 and 64 would be the most favorable selection in the present case.
6. Summary and Final Conclusion

6.1. Summary

At the very beginning of this thesis the importance of the early ship design phase is emphasized and station keeping is explained. Afterwards, the characteristics of the German offshore wind industry are pointed out. Based on this, the need for the purpose-built third generation of offshore wind farm installation vessels is explained and a set of design boundary conditions for this rather new ship type is listed. One of the major design tasks for these vessels is dynamic positioning.

The design of DP systems is focused upon in the second chapter. It is highlighted that not only static but also dynamic considerations of DP systems should already be done in the early design stage. This requires time series and mean values of the horizontal wave drift forces and the wave drift yaw moment.

In the main part of this thesis a calculation procedure for the computation of time series for the longitudinal and transverse wave drift forces as well as the wave drift yaw moment is presented. The focus of this procedure is on the computation of time series as input data for the dynamic DP approach. Based on the calculated time series the determination of time-averaged mean values is a side product. These mean values can be used as input data for a static DP analysis. Thus the presented computational procedure can be used for the computation of input data for the evaluation and design of DP systems following the static as well as the dynamic approach. The use of the strip theory in combination with the presented short wave correction makes the procedure fast and therefore applicable as a first principle design tool within the early ship design.

Following computed results of the horizontal wave drift forces are validated with measured data from wave basin tests for the SIETAS TYPE 187. The validation is done for three sea states. For each sea state five encounter angles are evaluated. Furthermore, four different variations of the frequency domain computations are used as basis for the determination of the times series for the horizontal wave drift forces. Finally, it is pointed out that in the presented case the accordance of the calculated to the measured values is most favorable from a ship design point of view, when neglecting the quadratic velocity term but applying the short wave correction.
6.2. Recommendations for Further Studies

In this thesis the validation of the computational procedure is only based on the data measured for one model. This is due to the fact that such measurements are rare and the owners of such information are not always willing to share their data. Even though wave basin tests are expensive, further validation of the presented calculation procedure is recommended. Furthermore, the validation with other ship types would also be interesting.

For the sake of completeness it has to be mentioned that waves, current and wind influence each other in reality. Hence the pure superposition of the different environmental forces for the use as input data for dynamic positioning is a theoretical model with some weaknesses.

The modular implementation of the computational procedure within the ship design environment E4 makes it flexible. If needed, it is possible to use frequency domain results from a different source than strip theory as input data for the time domain approach. Furthermore, the modular implementation does allow the further development to compute frequency domain results for different sea states. However, as shown above this is not really necessary. In addition, alternative short wave correction methods could also be implemented and tested. For this purpose it would be very interesting to do additional research concerning the question when a wave should be treated as a short wave.

The influence of the hull form over the static waterline is neglected by using strip theory and the presented short wave correction as well. It would be interesting to examine the influence of the over water part within the wave impact zone of the hull on the horizontal wave drift forces and the yaw moment.

However, for all improvements of the presented computational procedure it always has to be kept in mind that methods for the early ship design have to be fast. So it is always a trade-off between computational time and accuracy.

6.3. Final Conclusions

The presented computational procedure fulfills its goal of providing horizontal wave drift forces and yaw moments as input data for static and dynamic DP calculations. However, it is not only useful for the zero-speed dynamic positioning application. For vessels with forward speed the wave-induced additional resistance is, for engineering purposes, the same as the negative longitudinal wave drift force. Hence the presented computational procedure can be used, for example, for the determination of the course keeping ability
of a vessel in a seaway as well. Besides, the added resistance in a seaway is currently of large interest for shipyards and ship owners because of the Energy Efficiency Design Index (EEDI). The reason for this is the fact that a smaller installed main engine power yields to a better EEDI value. Thus direct calculations of the added resistance and therefore the resulting reduced installed main engine power may increase the competitiveness of a project. This may be achieved by using direct calculation results instead of the general consideration of the added resistance in waves by a sea-margin as a certain percentage of the required delivered power at the propeller in calm water.
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References


Appendix
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08.1987 - 07.1991 Grundschule Coldewey in Wilhelmshaven (Elementary School)
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10.2001 - 09.2002 Studies of business administration at Kiel University
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