COMPUTATION OF DRIFT FORCES FOR DYNAMIC POSITIONING WITHIN THE VERY EARLY DESIGN STAGE OF OFFSHORE WIND FARM INSTALLATION VESSELS

Philip H. Augener∗
Research Assistant
Institute of Ship Design and Ship Safety
Hamburg University of Technology
Hamburg, Germany
Email: Philip.Augener@tuhh.de

Stefan Krüger
Head of Institute
Institute of Ship Design and Ship Safety
Hamburg University of Technology
Hamburg, Germany
Email: krueger@tuhh.de

ABSTRACT

The German government has decided upon the changeover from fossil and nuclear based electrical power generation to renewable energies. Following from this offshore wind farms are erected in the exclusive economic zones of Germany. For the transportation and installation as well as the maintenance of the wind turbine generators very specialized vessels are needed. The capability of dynamic positioning even in very harsh weather conditions is one of the major design tasks for these vessels. For this reason it is important to know the external loads on the ships during station keeping already in the very early design stage. This paper focuses on the computation of wave drift forces in regular and irregular waves as well as in natural seaway. For validation the results of the introduced calculation procedure are compared to measured drift force data from sea-keeping tests of an Offshore Wind Farm Transport and Installation Vessel.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>Waterline angle regarding the longitudinal axis of the vessel</td>
</tr>
<tr>
<td>ω</td>
<td>Circular wave frequency</td>
</tr>
<tr>
<td>ρ</td>
<td>Density of sea water</td>
</tr>
<tr>
<td>µ</td>
<td>Wave encounter angle</td>
</tr>
<tr>
<td>L₁</td>
<td>Non-shadow part of the waterline</td>
</tr>
<tr>
<td>T₁</td>
<td>Average wave period</td>
</tr>
<tr>
<td>xₑ</td>
<td>Longitudinal coordinate of the center of gravity</td>
</tr>
<tr>
<td>yₑ</td>
<td>Tranverse coordinate of the center of gravity</td>
</tr>
<tr>
<td>x₀</td>
<td>Longitudinal coordinate of a waterline element</td>
</tr>
<tr>
<td>y₀</td>
<td>Transverse coordinate of a waterline element</td>
</tr>
<tr>
<td>ˆp</td>
<td>Complex amplitude of the pressure at a point on the mean waterline</td>
</tr>
<tr>
<td>ˆx</td>
<td>Position vector of a waterline element</td>
</tr>
<tr>
<td>ˆα</td>
<td>Complex amplitude of the vessel’s rotation vector</td>
</tr>
<tr>
<td>ˆx_G</td>
<td>Vector of center of gravity</td>
</tr>
<tr>
<td>ζ₀</td>
<td>Real wave amplitude of an elementary wave</td>
</tr>
<tr>
<td>ζ</td>
<td>Complex wave amplitude</td>
</tr>
<tr>
<td>ζ</td>
<td>Real wave amplitude</td>
</tr>
<tr>
<td>µ₀</td>
<td>Main wave propagation direction</td>
</tr>
<tr>
<td>ω_e</td>
<td>Encounter frequency between the ship and the waves</td>
</tr>
<tr>
<td>ω₀</td>
<td>Modal frequency of the specified seaway class</td>
</tr>
<tr>
<td>εᵢ</td>
<td>Random phase angle of irregular waves</td>
</tr>
<tr>
<td>εᵢₗ</td>
<td>Random phase angle of natural seaway</td>
</tr>
<tr>
<td>ˆYₚ</td>
<td>Complex transfer function of the pressure at a point at the mean waterline</td>
</tr>
<tr>
<td>ˆY₄</td>
<td>Complex transfer function of the roll motion</td>
</tr>
</tbody>
</table>

∗Address all correspondence to this author.
the German government. For the installation at first place and

generators (WTG) are required to fulfill the ambitious goals of
the German offshore wind farms. The areas for the erection of these wind
turbine farms are the exclusive economic zones of Germany in the North
Sea as well as the Baltic Sea. Several thousands of wind turbine
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the German government. For the installation at first place and

INTRODUCTION

The German politics have decided upon the energy
turnaround for sustainability, meaning the changeover from nu-
clear and fossil based electrical power generation to the use of
renewable energies. One major stakeholder for this switch are
offshore wind farms. The areas for the erection of these wind
farms are the exclusive economic zones of Germany in the North
Sea as well as the Baltic Sea. Several thousands of wind turbine
generators (WTG) are required to fulfill the ambitious goals of
the German government. For the installation at first place and

WAVE FORCES

When talking about wave forces on offshore structures and
ships it is important to distinguish between first and second or-
der forces. The first order wave forces are mostly relevant for
the construction of offshore structures as stated in [1]. With a
zero mean value averaged over one period, the first order forces
oscillate with the wave frequency. Wave drift forces, which are
second order wave forces, are about one magnitude smaller in re-
lation to the first order forces. With a non-zero mean value, wave
drift forces are proportional to the square of the wave amplitude
in harmonic waves. This leads to a constant timely weighted av-
erage resulting force in the propagation direction of the waves.
Hence wave drift forces have to be considered for station keep-
ing. In irregular waves drift forces are proportional to the square
of the envelope of the exciting waves. The reason for this is the
slow change of the wave drift forces regarding the variation of the
wave heights in irregular seas. With good approximation the lon-
gitudinal wave drift forces are equivalent to the added resistance
in waves for ships with forward speed, but normally defined with
an opposite sign.
drift moment of yaw is from an unpublished paper by Söding. The following equations for the longitudinal and transverse wave drift forces as well as for the wave drift moment of yaw is explained. The procedure is subdivided into two parts. The first part is the computation of the drift forces in the frequency domain in harmonic waves of unit wave amplitude. In the second part the computation in the time domain using irregular waves and natural seaway is explained. The described calculation procedure is integrated in the ship design environment E4, for which a detailed description can be found in [2] and [3].

**Frequency Domain Computations**

In 1970 Boese published his theory for the calculation of the added resistance in waves in the frequency domain [4]. In this theory the second order forces are determined based on first order results. For this approach the required first order motions of a ship are calculated according to strip method as described in [5] and the first order pressures on the hull surface are determined with the pressure strip method by Hachmann [6]. While normally the calculation of pressures is omitted in common strip methods, this is the focus of the latter method. In order to fit the usual strip theory the pressure strip method has been adapted in [7]. Based upon this the derivation of the following equations for the longitudinal and transverse wave drift forces can be found in [8], while the equation for the wave drift moment of yaw is from an unpublished paper by Söding.

Figure 1 shows the inertial coordinate system used for the description of the theory. The origin is located at the intersection of the midship section, the midship plane and the keel. Furthermore it has to be mentioned, that the system is rotated by 180° around the longitudinal axis for the output of the results. Following from this the output of the longitudinal forces is positive forward, the transverse forces are positive to portside and the output of the yaw moment is positive turning counterclockwise around the upward pointing axis. For the time-averaged longitudinal wave drift force Eqn. (1) is valid and for the time-averaged transverse wave drift force Eqn. (2) is used. Equation (3) shows how the time averaged wave drift moment of yaw is calculated. In Eqn. (1), Eqn. (2) and Eqn. (3) the forces on the hull of the vessel beneath the average still-water line are accounted for by the first summand, while the second summand considers the forces on the ship due to the relative motion between the vessel and the water surface. The third summand in Eqn. (1) is a correction term valid for ships with a non-wetted transom beneath the waterline at forward speed. In Eqn. (3) the index ζ indicates the relevant vector component for the moment of yaw and the last two terms are needed because the origin of the coordinate system is not the center of gravity.

\[
\begin{align*}
F = \frac{1}{2} \left( m \omega^2 \text{Re} \left( \hat{Y}_x \right) \right) \\
+ \rho g \sum_{\text{Str,Port}} \int_{L} |\hat{P}_{cr}|^2 \frac{dy^+}{dx} \, dx \\
+ \rho g \left( |\hat{Y}_a|^2 + |\hat{Y}_s|^2 \right) \\
\cdot \left[ A_s \left( T + z_0 \right) \right]_{\text{transom}} \xi^2
\end{align*}
\]

\[
\begin{align*}
\bar{F} = \frac{1}{2} \left( -m \omega^2 \text{Re} \left( \hat{Y}_x \right) \right) \\
+ \rho g \sum_{\text{Str,Port}} \int_{L} y^+_w \frac{dy^+}{dx} \, dx \xi^2
\end{align*}
\]

\[
\begin{align*}
\bar{M} = -\frac{1}{2} \omega^2 \text{Re} \left[ \hat{a} \times \hat{a} \right]_{\xi} \\
+ \sum_{\text{WL,Elements}} \left( \frac{|\hat{P}_{cr}|^2}{4 \rho g} \Delta_x \times \left( 0, 0, 1 \right) \right) \xi \\
+ \hat{x}_f \bar{F}_k - \hat{y}_f \bar{F}_k
\end{align*}
\]

Equation (1) is already used in the ship design environment E4 for the determination of the added resistance in waves as described in [9]. However the determination of the relative motion between the water surface and the ship is improved. While for the added resistance only the Froude-Kriloff forces, which
are caused by the undisturbed pressure field due to the incident wave, are used, the new implementation additionally accounts for the diffraction and radiation forces. Corresponding to [8] this is achieved by using the above mentioned approach by Hachmann [6] to compute the complex amplitudes of the pressures at the mean waterline. Based on this the complex transfer functions of the pressures are determined and with these the complex transfer functions of the relative vertical motion are found by Eqn. (4).

\[ \hat{y}_{cr} = \frac{\hat{F}_\rho}{\rho g} \]  

(4)

For the aforesaid way to determine the wave drift forces typical results are revealed in the following. Figure 2 shows typical results for the longitudinal and Fig. 3 for the transverse wave drift force, while in Fig. 4 characteristic results for the wave drift moment of yaw are displayed.

In the diagrams the encounter angles are counted counterclockwise from \( \mu = 0^\circ \) for following seas over the starboard side to \( \mu = 180^\circ \) for head seas and back over the port side of the vessel. For the symmetric model at an un-heeled floating position, symmetric results for both sides of the vessel have to be expected as well.

Faltinsen explains in [10], that wave drift forces for long waves originate from the motions of the vessel, while drift forces in short waves are mainly due to the reflection of the incoming waves from the ship’s hull. For the longer waves the above mentioned equations can be used, whereas the calculation for the zero speed case for short waves is done in accordance to [10] as explained below. Figure 5 shows the definitions used for the following equations. It has to be mentioned that as \( \beta = 0^\circ \) for head seas, it has to be transformed by \( \beta = \pi - \mu \) for the used angle of encounter \( \mu \), which is zero in following seas. Equation (5) shows how the longitudinal wave drift forces are calculated in short waves, while Eqn. (6) is valid for the transverse wave drift forces and Eqn. (7) for the wave drift moment of yaw.

\[ \overline{F}_{\xi \text{ short}} = \frac{\rho g \hat{\xi}_2}{2} \int \sin^2 (\gamma + \beta) \cdot \sin (\gamma) \, dl \]  

(5)

\[ \overline{F}_{\eta \text{ short}} = \frac{\rho g \hat{\eta}_2}{2} \int \sin^2 (\gamma + \beta) \cdot \cos (\gamma) \, dl \]  

(6)

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\[ \dot{M}_{y \text{ short}}^2 = \frac{\rho g \xi_0^2}{2} \int_{L_1} \sin^2(\gamma + \beta) \cdot (x_0 \cdot \cos(\gamma) - y_0 \cdot \sin(\gamma)) \, dl \] (7)

\[ \text{Enc.ang [deg]} \]   0.0   45.0   90.0   135.0   180.0   225.0   270.0   315.0   360.0

\[ \text{Driftforce in kN/m^2 wave amplitude} \]

\[ \text{Circular frequency [rad/s]} \]

\[ \text{Enc.ang [deg]} \]   0.0   45.0   90.0   135.0   180.0   225.0   270.0   315.0   360.0

\[ \text{Driftmoment in kNm/m^2 wave amplitude} \]

\[ \text{Circular frequency [rad/s]} \]

\[ \text{Enc.ang [deg]} \]   0.0   45.0   90.0   135.0   180.0   225.0   270.0   315.0   360.0

The results from the frequency domain calculations for different encounter angles with the unit wave amplitude, as described in the previous section, will be used as input data for the time domain computation described in this section. In the beginning a time domain computation procedure for the calculation of the wave drift forces and the wave drift moment of yaw in unidirectional irregular waves according to [1] is presented.
Based on this an enhancement for natural seaway is introduced in the second subsection.

**Irregular Waves.** Using the wave drift forces determined with the equations above as input values and combining them with the seaway data from energy density spectra, it is now possible to determine time dependent forces in irregular waves of arbitrary heights and with different encounter angles. Depending on the selected input data, Eqn. (8) is valid for the calculation of time series for the longitudinal and transverse wave drift forces as well as the wave drift moment of yaw as explained in [1].

\[
F^2(t) = \frac{\rho}{2} \cdot g \cdot L \cdot \alpha_0^2 \cdot a^2(t)
\]  

Equation (8) shows the dependency of the wave drift forces on the square of the envelope of the random seaway, with the envelope of the random seaway corresponding to the amplitude fluctuation of the random seaway under consideration. Furthermore the dependency on the mean drift force coefficient becomes clear, which is determined by Eqn. (9), which indicates that the mean drift force coefficient is a mean value weighted with the seaway defined by the energy density spectrum.

\[
\alpha^2_0 = \frac{\int_0^\infty \alpha^2(\omega) \cdot S_{zz}(\omega) \, d\omega}{\int_0^\infty S_{zz}(\omega) \, d\omega}
\]  

Equation (10) represents the definition of the drift force coefficient, either for the longitudinal or transverse wave drift forces as well as the wave drift moment of yaw. The equation shows how the drift force coefficient is computed from the frequency domain solution with \(\zeta^2_0\) being the square of the unit wave amplitude.

\[
\alpha^2(\omega) = \frac{F^2(\omega)}{\frac{\rho}{2} \cdot g \cdot L \cdot \zeta^2_0}
\]  

The typical generation of a time series of a random seaway from a seaway spectrum is presented in Eqn. (11). The wave spectrum is subdivided into \(J\) parts.

\[
\zeta(t) = \sum_{j=1}^J \sqrt{2 \cdot S_{zz}(\omega_j) \Delta\omega_j} \cdot \text{Re} \left( e^{i(\omega_j t + \epsilon_j)} \right)
\]  

Based on Eqn. (11) the square of the envelope of the random seaway can be determined with Eqn. (12).

\[
a^2(t) = \zeta^2(t) + \frac{(d\zeta / dt)^2}{\omega^2_0}
\]  

Typical results for the aforesaid calculation procedure for the longitudinal wave drift force are presented in Fig. 9 and for the wave transverse drift force in Fig. 10. Both time series clearly constitutes the non-zero mean value. Fig. 11 shows characteristic results for the wave drift moment of yaw, where the non-zero mean value can be noticed as well. All three presented examples
are extracted from time series with $10^4$ seconds simulation time with time steps of 0.5 seconds.

**Natural Seaway.** Adapted from the above described calculation procedure for unidirectional irregular waves from [1], this subsection deals with the calculation of time series for wave drift forces and the wave drift moment of yaw in natural seaway. Keeping in mind Eqn. (13), as stated in [11], and combining Eqn. (13) with Eqn. (9) this leads to Eqn. (14).

$$\int_0^{\infty} S_{zz}(\omega) \, d\omega \approx \sum_{j=1}^{J} \frac{\zeta^2(\omega_j)}{2}$$  \hspace{1cm} (13)

$$\alpha_0^2 = \frac{\sum_{j=1}^{J} \alpha^2(\omega_j) \cdot \zeta^2(\omega_j)}{\sum_{j=1}^{J} \zeta^2(\omega_j)}$$  \hspace{1cm} (14)

As described in [12] the complex wave amplitude for short crested seaway can be determined with Eqn. (15) by using Eqn. (16).

$$\hat{\zeta}(\omega_j, \mu_l) = \sqrt{2 S_{zz}(\omega_j, \mu_l)} \Delta \omega_j \Delta \mu_l \, e^{i \epsilon_{jl}}$$  \hspace{1cm} (15)

$$S_{zz}(\omega, \mu) = S_{zz}(\omega) \cdot \frac{2}{\pi} \cos^2 \left( \mu - \mu_0 \right),$$  \hspace{1cm} (16)

for $|\mu - \mu_0| \leq \frac{\pi}{2}$, else $= 0$.

In Eqn. (17) the relation between the real and the complex wave amplitude is shown. The real wave amplitude is equal to the absolute value of the complex wave amplitude.

$$\zeta(\omega_j, \mu_l) = |\hat{\zeta}(\omega_j, \mu_l)|$$  \hspace{1cm} (17)

Based on Eqn. (14) and using Eqn. (17) with Eqn. (15) and Eqn. (16) the mean drift force coefficient for natural seaway is derived according to Eqn. (18).

$$\alpha_0^2 = \frac{\sum_{j=1}^{J} \sum_{l=1}^{L} \alpha^2(\omega_j, \mu_l) \cdot \zeta^2(\omega_j, \mu_l)}{\sum_{j=1}^{J} \sum_{l=1}^{L} \zeta^2(\omega_j, \mu_l)}$$  \hspace{1cm} (18)

Based on the frequency domain results the wave drift force coefficients are determined according to Eqn. (19).

$$\alpha^2(\omega, \mu) = \frac{F^2(\omega, \mu)}{\frac{2}{\pi} \cdot g \cdot L \cdot \zeta^2}$$  \hspace{1cm} (19)

To generate the required time signal of the wave amplitude Eqn. (20) from [12] can be adopted.

$$\hat{\zeta}(t) = \sum_{j=1}^{J} \sum_{l=1}^{L} \sqrt{2 S_{zz}(\omega_j, \mu_l)} \Delta \omega_j \Delta \mu_l \cdot e^{i (\omega_j t + \epsilon_{jl})}$$  \hspace{1cm} (20)

Finally applying Eqn. (18), Eqn. (19) and Eqn. (20) instead of Eqn. (9), Eqn. (10) and Eqn. (11) the above described procedure for irregular waves can be used for natural seaway as well.

**VALIDATION**

For the validation of the presented calculation procedure sea-keeping test results for the Offshore Wind Farm Installation
Vessel Sietas Type 187 in unidirectional irregular waves are compared to computed results based on a model of the vessel in the ship design environment E4. The Sietas Type 187 is purpose build for the transportation and erection of offshore wind turbines and it is thus belonging to the third generation of this specific ship type. This generation of jack-up ships is especially designed and constructed according to the specific requirements of the offshore wind industry, while older jack-up vessels are mostly converted other ship types. The main reason why so much effort is put into these new vessels is the fact, that no offshore wind turbines shall be visible from ashore in Germany. This directly leads to installation sites far away from the coastline and hence to greater water depths and a very harsh environment for the operation of these vessels. The Sietas Type 187 is presented in Fig. 12 and the main characteristics of the vessel can be found in Tab. 1.

From this table the length/beam ratio can be identified as rather small and the beam/draught ratio as large in comparison to conventional ships. There are several boundary conditions leading to these unusual ratios: For the transportation of the wind turbines and foundations a large deck area is required, leading to a great beam of the vessel. Additionally the relatively small length and draught of the vessel are caused by the restricted water depth and berth length at the offshore terminals, where the vessel loads its cargo. Another reason for the rather small length is the longitudinal bending moment, which is critical because of the required jacking capability of the vessel, especially in combination with a rather small side depth. Furthermore it is necessary to enable a secure standing position for the vessel in its jacked-up position. This works best with a small length/beam ratio with the legs located as far away from each other as possible.

A critical examination concerning the applicability of the strip theory is necessary, because of the unusual main dimension ratios of the vessel and the fact that strip methods are based on slender body assumptions. However then the length/beam ratio

| TABLE 1. MAIN DIMENSIONS OF SIETAS TYPE 187. |
|---|---|---|
| Length | [m] | 139.40 |
| Beam | [m] | 38.00 |
| Draught (design) | [m] | 5.70 |
| Depth | [m] | 9.12 |
| Engine | [kW] | 4 x 2,500 |
| Deadweight (design) | [t] | 6,500 |
| Speed (design) | [kn] | 12.00 |
| Crane lift main hoist | [t] | 900 |
| (30 m outreach) | | |
| Maximum water depth for jacking | [m] | 45.00 |
| Length/Beam | | 3.67 |
| Beam/Draught | | 6.67 |
of the Sietas Type 187 is 3.67, being still significantly larger than the threshold value of 2.5 for which a remarkable effectiveness for the prediction of ship’s motion by strip theory is testified in [10]. In supplement the use of the strip theory is approved with good results for wave length as short as one third of the vessel’s length in [13], which leads to critical wave frequencies higher than approximately 1.1 rad/second for the vessel under consideration. Hence if the discretization of the strips is done properly, the large beam/draught ratio of the vessel should not be a problem.

The analysis described below is based on a loading condition of the vessel floating with even keel on its design draught. The utilized model in E4 consists of the buoyancy body and the ship’s displacement, considering the weight distribution in all three spatial directions for the correct representation of the mass moments and products of inertia. The complex transfer functions for all six degrees of motions and the pressures are computed based on equidistant discretized stripes. The sea-keeping tests have been performed with a model in scale 1:27 for the three sea states listed in Tab. 2. And for the simulation the modified Pierson-Moskowitz spectrum according to Eqn. (21) from [12] is used.

\[
S_{\text{z2}}(\omega) = 173 H_{1/3}^2 T_1^4 \omega^{-5} e^{-692 T_1^4 \omega^{-4}}
\]  

(21)

The relation between the peak period and the average period for a Pierson-Moskowitz spectrum can be found in [14] and it is defined as shown Eqn. (22).

\[
T_1 = 0.771 \cdot T_p
\]  

(22)

The wave drift forces in this section are simulated for \(10^4\) seconds, which is the recommended magnitude in [1] to achieve reasonable results. Fifty random wave components are used for the composition of the irregular seaway. And since the sea-keeping tests have been performed with an unidirectional seaway, no angular spreading is used. The short wave correction is applied for wave lengths smaller than the length of the vessel for the longitudinal and for waves shorter than half of the ship’s length for the transverse drift forces. The comparison between the measured wave drift forces and the computed results is done by the mean values of the time series of the forces, because these measurements were available only. No measurements for the wave drift moment of yaw have been carried out. For the seaway specified in Tab. 2 the comparisons of the measured data to the computed results are presented in Fig. 13 for seaway W4, in Fig. 14 for W5 and in Fig. 15 for W6.

The presented results are reasonable for all three sea states.

<table>
<thead>
<tr>
<th>Name</th>
<th>Spectrum</th>
<th>(H_{1/3}) [m]</th>
<th>(T_p) [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>W4</td>
<td>Pierson-Moskowitz</td>
<td>1.8</td>
<td>7.5</td>
</tr>
<tr>
<td>W5</td>
<td>Pierson-Moskowitz</td>
<td>1.8</td>
<td>11.3</td>
</tr>
<tr>
<td>W6</td>
<td>Pierson-Moskowitz</td>
<td>1.8</td>
<td>15.0</td>
</tr>
</tbody>
</table>
Especially the accuracy for the longitudinal wave drift force in following seas is good, while the mean value of this force in head seas is overestimated by the computation. The transverse wave drift forces are also overestimated by the presented calculation procedure. But remembering that wave drift forces are second order forces, which are computed based on first order results here, and assuming an accuracy of 10-15 % for the first order results, the accuracy of the second order results may not be better than 20-30 % as it is mentioned in [15]. Additionally conservative results are rather favorable for the design of DP-Systems.

CONCLUSIONS

In this paper a calculation procedure for the computation of time series for the longitudinal and transverse wave drift forces as well as the wave drift moment of yaw is presented. The use of potential theory makes the procedure fast and therefore applicable as a first principle design tool in the very early ship design stage. The comparison to sea-keeping test results leads to an acceptable accordace. Together with the simulation of wind forces as formulated in [16] the described procedure can be used for the generation of input data for a dynamic positioning code as introduced in [17].

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