ABSTRACT: In recent years, many slender structures were equipped with tuned mass dampers for vibration control. If properly tuned, these passive dampers can mitigate structural vibrations. Active mass dampers can be more efficient and used as an alternative. However, their application for high-rise buildings is very rare and — to the authors’ knowledge — no such devices have been installed yet in bridges. Several reasons can be identified. Besides issues concerning reliability, the main problem is to create an efficient active device which has a low self-weight, is simple in mechanical design, possesses a robust control scheme, has a low power demand of its actuators, and consumes little energy. In this paper, a new active mass damper is presented which possesses these properties, thus overcoming drawbacks of conventional active mass dampers. Furthermore, it is shown that by varying the basic concept and using combinations of the device, the damper can be applied to various problems. One possible application of the new device is the control of lateral vibrations of a bridge deck induced by pedestrians. A bridge model equipped with a prototype active mass damper was designed, built, and tested at the Structural Analysis and Steel Structures Institute of Hamburg University of Technology. The tests impressively demonstrated the feasibility of the proposed damper concept. With a ratio of the active mass to the vibrating mass of the model of 0.48 %, an equivalent viscous damping ratio of 5.6 % was obtained. Another example application involves flutter control of a bridge deck section model. In wind tunnel experiments performed at the authors’ institute, the critical wind speed was doubled with a mass ratio of 1.0 %. It can be concluded that the presented new active mass damper is highly versatile and efficiently reduces structural vibrations of various excitation sources.

KEY WORDS: Active mass damper; Active control; Experiments; Pedestrian induced vibrations; Flutter.

1 INTRODUCTION

One method to mitigate structural vibrations is to apply active control [1]. Conventional active mass dampers generate control forces by accelerating and decelerating auxiliary masses. An actuator, frequently an electric motor, provides the necessary acceleration of such a control mass. The driving power depends on this acceleration and its maximum is generally larger for larger required control forces. Thus, the performance of a conventional active mass damper is limited by the capacity of the installed actuator.

The acceleration of the control mass is associated with an increase of the mechanical energy of the control mass. Mechanical breaking power is needed to decelerate the control mass, thereby decreasing its mechanical energy. Consequently, during the continuous process of accelerating and decelerating the control mass, energy is cyclically input into the system and extracted again from the system. The effect of damping the structure results from the fact that the extracted energy is larger than the energy input into the system. Usually, the extracted energy is dissipated in the active device itself and no additional device that dissipates the energy is required. To increase the energy efficiency of a conventional active mass damper, the extracted energy could be reused. For this, it is necessary to store the energy, for example, mechanically or electrically. This, however, requires an additional devices in the damping system which increases the initial costs and may be inefficient due to energy losses.

By utilizing a force generation mechanism based on the motion of a control mass with constant velocity, an active mass damper can be created that requires significantly less actuator power compared to conventional active mass dampers. Moreover, the cyclic energy transfer is avoided.

2 DAMPER CONCEPT AND POSSIBLE CONFIGURATIONS

2.1 Basic unit of twin rotor damper

The basic idea is to utilize centrifugal forces generated by pairs of eccentrically rotating masses (unbalances) in such a way that undesired force components cancel. The proposed new device is termed “twin rotor damper”, and one configuration is shown in Figure 1.

Figure 1. Basic damper unit generating resultant vertical force.
Two rotors rotate with the same angular speed \(\omega\) in opposite directions around horizontal axes (Fig. 1). Each rotor consists of an actuator-driven rotating rod with length \(r_c\) and a mass \(m_c\) attached to its free end. The rotors perform complete revolutions and both point in the upward direction at the same instant of time and in the downward direction at another point in time. The generated centrifugal forces \(F_R\) are used for the control of the supporting structure. Since the horizontal force components \(F_h\) cancel at all times, solely a resulting vertical force \(F_v = 2F_v\) is generated which can be used for damping vertical vibrations. \(F_v\) is the vertical force component of the centrifugal forces generated by one rotor. With a constant speed of rotation \(\omega\), the generated force is harmonic with circular frequency \(\omega\). The amplitude of this dynamic force is given by

\[
F_c = 2m_c r_c \omega^2
\]

where it is assumed that the rod is massless and the distance between the center of gravity of a control mass and the center of the associated rotation axis is \(r_c\).

2.2 *Generation of a directed force*

The direction of the generated control force can be varied by changing the relative position between the two rotors. Since the rotors rotate in opposite directions with the same speed, they are necessarily aligned parallel to each other twice per completed revolution. The direction of the generated force is parallel to these specific rotor positions as shown in Figure 2. Accordingly, a horizontally acting resultant force is generated if both rotors point periodically in the same horizontal direction (Fig. 3). Hence, it is possible to generate control forces with diverse directions using the same device. Independently of the direction, the amplitude of the generated force (Eq. 1) remains the same.

It is noted that an additional harmonic moment develops if non-vertical forces are generated (Fig. 2). There are two sources for this moment. First, the moments resulting from the weight of each control mass and the corresponding vertical projection of the rotor length do not cancel. Second, due to the offset of the rotor axes in the direction of the generated force, the dynamic force creates a moment. Furthermore, the latter moment is proportional to the distance between both rotor axes. A detailed evaluation of the moments can be found in [2].

The development of a moment caused by the dynamic force can be prevented by avoiding an offset of the rotor axes in the direction of the generated forces. Another possibility is to attach both rotors to the same axis. In this case, a design with compact dimensions is obtained. Since the rotors rotate in opposite directions and exhibit the same position twice per revolution, the control masses must possess an offset in the direction of the jointly used rotor axis (Fig. 4). To avoid a dynamic moment perpendicular to the rotor axis, one mass can be divided into two equal parts, and the associated rotors are placed with equal distances to the rotor with the undivided mass (Fig. 5).

As an alternative to adjusting the direction of the generated force by influencing the relative rotor position, the direction can be adapted by using the configuration of Figure 1 and modifying the inclination of the rotor axes. In this way, no additional moments perpendicular to the plane that contains the rotating masses are generated. Following this approach, a pure
horizontal control force is generated by aligning the rotor axes in the vertical direction. This layout is especially attractive for the control of vibrations in the horizontal direction because the control masses do not need to be lifted during control. Consequently, no actuator power is required for the generation of a harmonic horizontal control force provided that the support of the device does not move and losses, e.g. due to friction, are neglected. A possible application of an active damper in this configuration for the control of lateral vibrations is studied in the first example given below.

Although in the configurations with non-vertical axes actuator power is needed to lift the control masses, no power is required for the generation of the harmonic control force. The kinetic energy of the control masses remains constant, whereas the potential energy varies.

Depending on the position of the damping device relative to the structure, the generated control force may induce moments in the structure. For example, if the force vector does not intersect the elastic axis of a controlled line-like structure a torsional moment is induced which has to be considered in the design of the structural control system.

2.3 Exclusive generation of a moment

The exclusive generation of a control moment is possible by slightly varying the basic design of the damper. If, as indicated in Figure 6, both rotors rotate in the same direction and their relative position is such that the rotors point in opposite directions, the resultant vertical and horizontal dynamic forces are always zero and, with a constant rotor speed, a harmonic moment with an amplitude of

\[ M_c = m_c r_c c_r \omega^2 \]

(2)

is generated. Thus, the moment is proportional to \( c_r \) which is the horizontal distance between the parallel rotor axes (Fig. 6).

The direction of the generated moment can be controlled by an appropriate inclination of the rotor axis. Furthermore, both rotors can be mechanically coupled with a transmission. This has the advantage that the rotors are steadily in static equilibrium even in the case of non-vertical axes. Hence, no actuator power is required for lifting the eccentric masses during control which also holds for the generation of the harmonic moment. A second application using this damper layout is presented below.

2.4 Combinations of damper units

Configurations of the presented damper can be combined to yield additional advantageous designs of the damping devices. The combination of two identical basic damper units (Fig. 1) described in Section 2.1 is particularly useful. Arranging both units in the same vertical plane with all rotor axes located in a perpendicular horizontal plane gives the straightforward opportunity to generate a vertical control force only, a control moment only, or certain combinations of a simultaneously acting vertical control force and a control moment.

Figure 7 shows both rotor units jointly generating a vertical control force. Using identical rotor speeds, each unit generates a vertical force with identical amplitude and phase. By simply changing the phases such that the phase of one force is opposite to the other one, a moment proportional to the distance between both centers of the units is exclusively generated (Fig. 8). Therefore, it is easily possible to switch from the control of vertical vibrations to the control of torsional vibrations. Moreover, by properly selecting the phase difference between the forces of both units, a dynamic vertical force and a moment with the same frequency is generated [2]. The force and the moment exhibit a defined phase difference. To achieve one such phase difference, there are two possible values for the phase difference between the individual vertical forces from the units which give two different combinations of force amplitude and moment amplitude.

In the example depicted in Figures 7 and 8, the active damper is installed inside a box girder of a long-span bridge which protects the device from environmental influences. The damping device allows to control vertical vibrations of the bridge deck induced by e.g. traffic or torsional vibrations that may result from wind. Results of numerical simulations for the latter application are given in [2] for flutter control of a long-span bridge and in [3] for control of buffeting induced bridge deck vibrations including the self-excited forces.

A modification of the orientation of the two identical damper units gives the opportunity to generate optionally one of two forces with mutually perpendicular directions within the plane
of the damper units. For this, the two units are aligned such that the plane containing the two rotor axes of one unit is perpendicular to the plane containing the rotor axes of the other unit. Operating both damper units either in phase or opposite in phase yields solely a resulting force that acts with a direction of 45° to one of the perpendicular planes. Hence, the generation of either an exclusive lateral control force or an exclusive vertical control force is also possible with a single device.

2.5 Further variants

The amplitude of any generated force is linearly dependent on the product of the control mass and the length of its rotor. Accordingly, various combinations of masses and rotor lengths yield identical force amplitudes. Thus, the weight and dimensions of the damping device can be adjusted depending on the available space and load capacity of the structure to be controlled.

The use of different masses or rotor lengths within a damper unit is possible; however, to avoid unfavorable resultant forces, the variation of these parameters may be restricted to differing units while the parameters within a single damper unit are constant. A variant of this configuration is studied in [2].

The basic unit can be altered and equipped with more than two rotors with different rotor orientations and axis alignments to create other variants of the proposed damper generating favorable forces and force combinations. Furthermore, additional versions of the damper are easily derived by suitably arranging and combining basic damper units and combinations thereof.

It is concluded that the concept of the proposed damping device is highly variable and adaptable to yield control forces specifically required in control applications. Therefore, the versatile device could be used in diverse structures susceptible to vibrations. Two example applications are given further below.

3 CONTROLLING THE AMPLITUDE OF THE GENERATED FORCE

3.1 Mechanical devices

As explained above, the damping device operated with constant speed \( \omega \) generates a harmonic force with an amplitude proportional to the control mass \( m \) and rotor length \( r \) (Eqs. 1 and 2). To modify the amplitude of the force in practical applications, a mechanical device that changes the length of the rotor in the plane of rotation is feasible, whereas a variation of the mass during control is hardly possible. Thus, one possibility to adapt the amplitude of the generated force during control is to vary the rotor length. An appropriate mechanism is, for example, the one used in a scissor jack.

Another possibility is to tilt the rotors around a pivot point located on the rotor axis. Depending on the location of the pivot point and tilting direction, the rotor describes a cone, a double cone, or a frustum of a cone. Considering the plane perpendicular to the original orientation of the rotor axis, the mass has in all cases a reduced distance to the rotation axis. This distance defines an effective rotor length. Similarly, the pivot point can also be located on the rotor dividing the rotor into two parts.

Tilting the axes of the damping device provides another means to vary the amplitude of a generated force in the original plane of rotation or of a generated moment perpendicular to the original rotation plane. However, the inclination of the force vector implies that a force component arises which may adversely affect the structure. Consequently, this method is only suitable for structures with large stiffness in the direction of the additional force component and which can transfer this force to the supporting structure.

For example, considering the configuration in Figure 7, the amplitude of the force acting vertically can be modified by tilting all rotor axes around a horizontal axis located in the plane containing the rotors. The horizontal projection on the shown vertical plane gives a vertical force with reduced amplitude while the vertical projection on the horizontal plane reveals a horizontal force which acts axially on the bridge deck. It can be assumed that the latter force is negligible since it is small compared to common axial loads in the bridge deck, and the bridge deck provides sufficient axial stiffness and mass.

In general, parallel rotor axes do not need to remain parallel; tilting the axes with respect to each other may be advantageous.

The amplitude of a moment that depends on the distance between damper units can be controlled by simply changing this distance. In the example depicted in Figure 8, one or both units can be displaced horizontally to modify the amplitude of the generated moment.

3.2 Modifying the phase between two forces

Another way to control the amplitude of a force generated by a rotor damper is to use a second, possibly identical, damping device that generates a force with frequency and direction identical to those of the force generated by the first device. The amplitude of the resultant force which is jointly generated by the two devices can be set by adjusting the phase difference between the forces generated by each device. The amplitude of the resultant force is maximum if this phase difference is zero whereas the amplitude is minimum if a phase difference of 180° is applied. The use of two identical devices allows to generate a resultant force with small amplitudes; in fact, an amplitude of zero is obtained if the phase between the individual forces is 180°. Furthermore, the maximum amplitude of the resultant force is twice as large as that of one device. The exclusive generation of a resultant force in the desired direction is possible by using two properly aligned basic damper units (Fig. 1).

3.3 Modifying the directions of two forces

Considering two identical rotor dampers as arranged in Figure 7, gives an alternative in which the phases of the individual forces remain identical. In the given configuration, a resultant vertical force with twice the amplitude of either individual damper force is generated. By modifying the directions of both individual forces such that the angle between the direction of an individual force and the horizontal plane is equal but opposite to those of the other force, a resultant force with reduced amplitude is obtained. Figure 9 shows one of two options, the other is obtained by applying inwardly directed forces. In either case, the right half of the system is a mirror image of the left half. The resultant force exclusively acts in the vertical direction.
because both horizontal components of the individual forces cancel. Furthermore, the moment of one unit resulting from the offset in the direction of the force (compare Fig. 2) is canceled by the moment of the second unit.

The amplitude of the resultant vertical force becomes zero when the directions of the individual forces are horizontal. Then, no resultant force is generated because the forces generated by each unit are equal with opposite directions.

The amplitude of a moment jointly generated by two rotor units (Fig. 8) can be modified in a similar way. In contrast to the previous case, the modified directions of the individual forces are such that they are parallel and opposite, i.e. the force vectors are rotationally symmetric. Again, there are two possibilities to obtain a particular moment amplitude, one is shown in Figure 10. In either case, neither a resultant vertical nor a resultant horizontal force is generated.

The method can also be used in other configurations of the proposed damper. Moreover, arranging all rotor axes along a single axis allows a compact design for dampers generating linear forces. Moments perpendicular to that axis can be avoided by dividing masses as described above and depicted in Figure 5. Figure 11 shows an appropriate device for the generation of a resultant vertical force with variable amplitude. By re-sorting the rotors in this example, it can be seen that the concept of modifying the directions of the individual forces generated by rotors forming a unit (described in this section) and the concept of modifying the phase between two forces (Sec. 3.2) generated by units formed from the re-sorted rotors merge (Fig. 12) when all rotor axes are arranged along a single axis.

4 APPLICATIONS

4.1 Particularly suitable fields of application

As described above, the proposed damper generates a harmonic control force if the rotor speed is constant. Different time histories of the control force can be generated by varying the rotor speed. However, to profit from the low power demand when generating harmonic control forces, applying the damper to structures which are optimally or nearly optimally controlled by a harmonic force is more expedient. Such structures vibrate uniformly and can be excited by various sources. Applications, therefore, include the control of vibrating structures with well-separated natural frequencies excited by a force or displacement with single frequency. The excitation can be due to, e.g., supported machinery, traffic, or pedestrians.
Excitations with a broad frequency spectrum such as wind, earthquakes, and waves can cause structural vibrations that vary strongly with time. A control with harmonic forces may not be effective. However, structures that exhibit characteristics of a low-pass filter often vibrate in a few modes that can be controlled by individual rotor dampers generating harmonic forces. Examples of such structures are long-span bridge decks, pylons, high-rise buildings, towers, cranes, and offshore structures. The continuing trend to ever more slender structures increases the potential for applications. Serviceability issues (e.g., comfort) or fatigue will play an increasing role for which active control is appropriate.

The energy efficiency of the proposed device is especially beneficial in applications with long duration and slowly varying conditions such as wind events with moderate turbulence intensity.

4.2 Specially developed control strategy

Modern control theory provides various strategies to generate control input for the actuator. However, in the examples below, a newly developed, straightforward approach is used which allows the application of the twin rotor damper without controlling the amplitude of the generated force. The implemented control also proves to be very robust.

The two presented examples represent uniformly oscillating structures which can be generalized to systems with one or two degrees of freedom. In the latter case, the coupled motion can be controlled by a single control force. Thus, a single damping device generating a single control force can be used in both examples.

Neglecting structural damping, it can be shown that a harmonic control force optimally damps a degree of freedom of the system if the control force is opposite in phase to the velocity of that degree of freedom. Furthermore, this is approximately true for systems with small viscous damping ratios or in the presence of a small harmonic excitation. Clearly, the damping effect of the control force on the system is reduced if the phase deviates from the optimum. For certain, determinable phases, the control yields vibrations with constant amplitudes. For phases of a certain range, the rotor damper excites the structure. This reflects a typical behavior of active control.

Based on these findings, an algorithm is developed which adjusts the phase between the generated control force and the controlled motion of the structure such that the vibrations are optimally damped. The use of the optimal phase ensures the best possible performance of the given damper layout. However, the action of a control force applied with the optimal phase can be unfavorable: the force excites the structure when the amplitude of the vibration is sufficiently small. To avoid this adverse effect, the efficiency of the control force is reduced by tuning to a non-optimal phase value, which depends on the system state and can be computed online. Alternatively, the control device can be disabled if the amplitude of the vibration falls below a threshold.

Accordingly, an algorithm is applied that controls the phase between the generated control force and the motion of the controlled structure. Deviations between the target value and the actual value of the phase are compensated by accelerating or decelerating the rotors for a predicted time and subsequently resuming the rotor velocity that matches the circular frequency of the vibration. Since these deviations are usually small, the accelerations can be limited. As a consequence, little actuator power is required for maintaining the phase or adjusting to a new phase value, i.e., for controlling the phase.

In the described control strategy, the amplitude of the generated control force remains constant. Hence, it is not necessary to adapt the amplitude of the force with one of the methods described in Section 3. The device can be used in its simpler configurations (e.g., Fig. 1, Fig. 6), as it is done in the examples below. The use of a simple mechanical design and a straightforward control algorithm makes the rotor damper an attractive active mass damper.

Further details are omitted here due to space limitations and will be presented elsewhere.

4.3 Example: control of horizontal vibrations

A model equipped with a prototype rotor damper was designed and built at the Structural Analysis and Steel Structures Institute of Hamburg University of Technology (Fig. 13). Figure 14 shows a rendering of the model. The model represents a horizontally oscillating structure such as a bridge. A possible application is the control of lateral vibrations of a bridge deck induced by pedestrians.

For the control of horizontal vibrations, the rotor damper is used in the basic configuration of Figure 1 with the rotor axes aligned in the vertical direction. The degree of freedom \( x \) of the model is the horizontal direction that is perpendicular to a line intersecting both rotor axes (Fig. 15). The main purpose of the tests is to demonstrate the proper functioning of the new damping device as well as of the proposed control algorithm.

The model of the bridge deck consists of a simple, ballasted girder suspended from a stiff frame by four thin steel cables (Fig. 14). Because the model acts as a pendulum, the natural cyclic frequency of vibration \( f_n \) can be set by adjusting the length of the cables. In the experiments of the presented results, \( f_n = 1.0 \text{ Hz} \). The model can freely vibrate: the direction perpendicular to the degree of freedom is not constrained. This

Figure 13. Test setup with twin rotor damper for control of horizontal vibrations.
Figure 14. Rendering of tested model.

Figure 15. Single-DOF system with twin rotor damper for control of horizontal vibrations.

results in low structural damping. For the considered degree of freedom, a damping ratio of $\zeta = 0.16$ was determined from experiments.

Considering the single-degree-of-freedom system in Figure 15, the equation of motion is given by

$$\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 2m_c\dot{r}_c\left(\phi \sin \phi + \phi^2 \cos \phi\right) \tag{3}$$

where $m = 124 \text{ kg}$ is the mass of the main system excluding the rotors; $c = 4\pi f_n \zeta (m + 2m_c)$ is the structural damping coefficient; $k = 4\pi^2 f_n^2 (m + 2m_c)$ is the structural stiffness; and $\phi$ is the rotor position. The dot indicates differentiation with respect to time $t$. Taking into account the mass distribution of the rotors, the damper parameters are $m_c = 0.296 \text{ kg}$ and $r_c = 0.181 \text{ m}$. Thus, the ratio of the total control mass to the total vibratory mass is $\mu_c = 2m_c/(m + 2m_c) = 0.48\%$.

Following the strategy proposed in Section 4.2, the rotor position $\phi$ is controlled using a specially developed MATLAB script. For this, the displacement $x$ is continuously measured with two laser displacement transducers and the mean value is monitored. If this value becomes larger than a threshold, a counter-rotating rotor motion with constant speed $\dot{\phi} = \omega = 2\pi f_n$ is initiated. Each rotor is driven by a separate electric motor. The motors are coupled in a master-slave configuration. Hence, only the master is controlled with the algorithm. As soon as an extremum of the displacement $x$ is detected, the actual rotor position is determined from the detected motor position of the master and compared with the rotor position associated with a control force with a phase opposite to that of the velocity $\dot{x}$. If the difference between both positions is significant, it is compensated using a moderate constant rotor acceleration ($2\pi$ to $8\pi \text{ rad/s}^2$). The rotor motion is stopped when the amplitude of the displacement $x$ falls below a predefined threshold.

Various tests using different configurations (e.g. rotor lengths, natural frequencies) were performed. Electromagnets were used to release the model from defined initial positions. Alternatively, the motion of the model was initiated by displacing the model by hand. The rotor speed was either predefined or identified from the measured vibration data during control.

Figure 16 shows a sample of an uncontrolled and controlled response. While the displacement without active damping decays only slightly during the selected time segment, the active damper significantly decreases the displacement. The amplitudes of the controlled motion decay nearly linearly with time. An equivalent viscous damping ratio can be determined by comparing the controlled displacement of the figure with the response of a system with only viscous damping. Using a viscous damping ratio of 5.6 % in this system yields the same displacement amplitude at the end of the shown time segment provided that the initial displacement is identical to that of the actively controlled system. Smaller initial displacements yield even larger equivalent viscous damping ratios.

The presented results can be directly transferred to an application in footbridges for the control of pedestrian induced lateral vibrations because the frequency and the motion amplitudes are typically similar. Hence, a significant mitigation of the induced vibrations can be achieved with the proposed device. Moreover, increasing the rotor lengths in the application allows a corresponding reduction of the control masses.

Additional tests with the model subjected to a continuous excitation are currently being prepared.
4.4 Example: control of bridge deck flutter

A model with a prototype rotor damper in the configuration of Figure 6 was designed and built at the authors’ institute. Figure 17 shows the bridge deck section model in the institute’s wind tunnel for flutter tests. Figure 18 depicts a rendering of the model excluding the suspension.

The motion of the model can be described with two degrees of freedom: heaving and rotation. In the presence of the air flow, the two degrees of freedom are coupled by self-excited aeroelastic forces. Due to this coupling, the combined motion can be damped with the exclusive action of a rotor-generated control moment vectored in the direction of the rotation. Analogously to above, an automatic control is applied based on the phase between the generated moment and the angular velocity of the model extracted from measurement. Since the frequency of the rotation varies with the wind speed, the momentary circular frequency is continuously identified and specified as the rotor speed.

Figure 19 shows identified critical wind speeds of the uncontrolled system and the controlled system, which were obtained with a mass ratio of $\mu_c = 1.0\%$ and an effective rotor length of $r_c = 0.052$ m. It can be seen that flutter is prevented if the amplitude of the dynamic angular displacement is sufficiently small, i.e., if the control is initiated at low dynamic angular displacements of the bridge deck model. In this case, torsional divergence becomes the governing mode of failure.

Being a static aeroelastic mode of instability, it cannot be controlled by any active mass damper.

5 CONCLUSIONS

The paper presents a versatile active mass damper which can be used for the control of structural vibrations of various excitation sources. As demonstrated in two separate experiments, the twin rotor damper effectively reduces vibrations requiring only low mass ratios. Employing a specially developed control algorithm allows the application of damper configurations with simple mechanical design.

The rotor damper is highly efficient. Contrary to conventional active mass dampers, the presented damper utilizes a motion with constant velocity leading to a low actuator power demand. In the damper configurations of the experiments, no mechanical power is required for the generation of a harmonic horizontal force or a harmonic moment.

Videos of both tests can be found at http://www.tuhh.de/sdb/.

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