Robustness assessment of a cable-stayed bridge

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ABSTRACT: The failure of one structural element can lead to the failure of further structural elements and thus to the collapse of large structural sections or the entire structure. Such disproportionate collapses have been discussed and investigated for some years, but mainly for buildings. Generally, structures can be made collapse resistant by ensuring a high level of safety against local failure or by increasing their robustness. Robustness has been defined as the insensitivity to local failure. Increasing the robustness of cable-stayed bridges means to design for the loss of cables. The sudden loss of cables is a dynamic event and must be investigated in non-linear dynamic analyses. For the design, only the maximum responses are of interest. For this, quasi-static analyses using dynamic amplification factors to account for the dynamic effects are advised. This paper examines the structural response of a cable-stayed bridge to the loss of one cable by means of dynamic analyses including large displacements. The effects of cable sag, transverse cable vibrations and structural damping are determined and dynamic amplification factors are computed. Finally, conclusions are drawn as to the collapse behavior of cable-stayed bridges taking into account non-linear material behavior.

1 INTRODUCTION

The failure of one structural element can lead to the failure of further structural elements and thus to the collapse of large structural sections or the entire structure. In many cases, the initial triggering event and the resulting damage are disproportionate. Such collapses have frequently been discussed and investigated in recent years and are generally summarized under the term progressive collapse. But work in this field refers predominantly to buildings.

With regards to the design of a structure against progressive collapse, the general aim is to ensure collapse resistance which means insensitivity to accidental circumstances. This can be achieved by ensuring a high level of safety against local failure or by using a design which allows for local failure. The structure's property of being insensitive to local failure is termed robustness (Starossek 2006b). The size of the local failure has to be defined by the design objectives. For cable-stayed bridges, collapse resistance is primarily achieved by increasing the robustness. The loss of cables must be considered as a possible local failure since the cross sections of cables are usually small and therefore possess a low resistance against accidental lateral loads stemming from vehicle impact or malicious action. Current recommendations for cable-stayed bridges constitute the sudden loss of one single cable (PTI 2001, fib 2005); other authors assume the sudden loss of all cables in a 10 m range (Starossek 2006a). The loss of cables can lead to overloading and rupture of adjacent cables. A collapse progressing in such a way is called a zipper-type collapse. Because the stiffening girder is in compression and a cable loss reduces its bracing against buckling, the collapse is accompanied and reinforced by a progressive destabilization of the structure (Starossek 2007). To trace the collapse progression following the rupture of one or more cables, dynamic non-linear analyses in the time domain are necessary.
This paper examines the dynamic response of a cable-stayed bridge to the loss of one cable. Effects of cable sag, transverse cable vibration and structural damping are determined in non-linear dynamic analyses taking into account large deformations. These kinds of analyses require a great amount of expertise and modeling effort. When designing for the loss of a cable, only the maximum responses are of interest. Quasi-static analyses using dynamic amplification factors to account for the dynamic effects can be conducted instead (PTI 2001). These dynamic amplification factors are determined here and the limits of such an approach are outlined. Finally, conclusions are drawn as to the collapse behavior of cable-stayed bridges. Ultimate states are calculated taking into account large displacements and non-linear material behavior.

2 BRIDGE SYSTEM AND MODELING

The cable-stayed bridge being considered is shown in Figure 1. Two cable planes are placed in a modified fan arrangement with 80 cables in each plane and a cable spacing of 15 m at deck level, apart from the closely spaced outermost back-stay cables. The cables have a total cross-sectional area of between 40.8 cm$^2$ and 117 cm$^2$. They are pre-tensioned in such a way that under dead load, the stiffening girder is not deflected at the cable anchorage points and the moment distribution corresponds to that of a continuous beam on rigid supports. The bending moments in the pylon are nearly zero under dead load. The pylons are made of reinforced concrete. The stiffening girder consists of a 21.60 m wide orthotropic deck, two 2.6 m deep longitudinal girders and cross girders spaced at 3.75 m. This corresponds to a slenderness ratio of 1/230. In longitudinal direction, the stiffening girder is only restrained by the cables.

![Figure 1: Structural system.](image)

The numeric investigation is conducted using a three-dimensional model of the bridge. The pylon and stiffening girder (longitudinal and cross girders) are modeled with beam elements. The modeling of the cables is described in Section 3. In addition to the dead loads, two live load cases are here considered: live load on the main span and live load on the side spans. For the loss of a cable, in accordance with the PTI Recommendations (2001), the live load is reduced to 75%.

The loss of a cable is investigated by non-linear dynamic analyses in the time domain, taking into account large deformations. Thus, direct time integration is used to solve the equation of motion. Firstly, the static initial state of the structure is calculated. The cable to be considered for failure is eliminated in the structural model and the corresponding cable forces (from dead load or from dead load plus live load) are applied to the anchorage nodes of that cable as static loads. The time-history analysis is begun on this modified and loaded system at rest. To model the sudden loss of the cable, a step impulse loading of the same size as the static cable force but acting in opposite directions is applied to both anchorage nodes. The numeric calculation is conducted using the finite element analysis program ANSYS.

3 INFLUENCE OF CABLE MODELING

The sudden loss of a cable leads to a redistribution of the load from the lost cable to the adjacent structural elements and to structural vibrations. The end nodes of the adjacent cables experience relatively strong dynamic displacements. The cable sag leads to a non-linear relationship be-
between these displacements and the corresponding cable forces. For static loading and small changes in the cable force, this non-linear relationship can be approximated by a tangential stiffness for an initial cable force. From this, an equivalent elastic modulus can be derived.

To evaluate the impact of cable sag and transverse cable vibrations in a dynamic cable loss scenario, a bridge model with simplified cable modeling is compared to a bridge model with realistic cable modeling. For the first, simplified model, each cable is represented by one prestressed tension-only truss element with a constant equivalent elastic modulus. The equivalent elastic modules and the cable forces are determined iteratively for the initial dead load condition. The cable’s mass and self-weight are assigned equally to both end nodes. For the second model, the cables are modeled realistically with sag and distributed masses along the length. A series of prestressed truss elements is used to model a cable. This initially kinematic system is loaded with its self-weight. Large displacements occur until a cable sag is achieved which is compatible with the loading and the pre-determined cable end forces. The number of truss elements is chosen such that the first ten natural frequencies of the vibrations transverse to the cable chord can be reproduced fairly accurately. In contrast to the first model, in this model, the dynamic properties as well as the non-linear force-deformation behavior of the cables are taken into account. Both models are adjusted in a way that virtually identical state variables under self-weight are produced in the intact condition. The loss of each cable is simulated as described in Section 2, and the extreme responses are determined from time-history analyses.

The extreme deflections of the stiffening girder are generally higher when the cables are modeled with equivalent truss elements instead of using realistic modeling (Fig. 2). The same applies to the extreme bending moments. They can be more than 40 % higher than those of the realistic model. Concerning the cable forces, the type of cable modeling is of minor influence. The truss element model leads to no more than 4 % higher values. The compressive forces in the stiffening girder also remain virtually unaffected by the type of cable modeling. Also, the extreme bending moments in the pylon are generally higher when the cable sagging is neglected. They can be more than 50 % higher than in the realistic model. However, locations exist where the pylon bending moments are up to 12 % lower in the simplified model compared to the realistic model.

The main reasons for the discrepancies outlined above are not the different static properties but the different dynamic properties of the two models. The different distribution of masses and the differing dynamic cable stiffnesses lead to shifts in the natural frequencies between the two structural models. In the simplified model, due to the concentrated cable masses at the cable

![Figure 2. Extreme vertical deflections of the stiffening girder due to permanent loads and loss of second longest cable at bridge center](image-url)
end nodes at the bridge deck, the excited deck frequencies are lower, and therefore the vibration periods are longer. Furthermore, the realistic model exhibits more modes. Here, deck vibrations are coupled with cable vibrations. In the realistic model, the loss of a cable excites more natural modes than in the simplified model. The superposition of the differing modes with their differing frequencies leads to different time histories where the peak responses do not necessarily occur when the vibrations begin, nor do they occur simultaneously. The amplitudes in the realistic model are generally smaller which might be explained by a phenomenon which is sometimes called “system damping” or “dynamic resistance” (Starossek 1992) but for the sake of brevity, is explained elsewhere.

4 INFLUENCE OF DAMPING

For long-span bridges with a steel deck and steel pylons, only small energy dissipations can be assumed. Values substantially smaller than 1% of the critical damping are given for cable-stayed bridges in the literature. In the present case damping ratios between 0.2 % and 1 % are considered. The analysis is based on the system described in Section 2, with the cables modeled as equivalent truss elements (Section 3). Rayleigh damping is assumed for the analysis because it is the only type of damping used in ANSYS when using direct time integration which is appropriate in this case. For determining the damping coefficients, it must be kept in mind that, for the loss of short cables, high modes (here, the 170th mode) significantly contribute to the structural response.

As stated in Section 3, the peak responses of the undamped cable-stayed bridge after cable loss do not necessarily occur within the first vibration period. Since amplitudes in damped systems are progressively reduced with time, the reducing effect of damping is dependent on the time of the occurrence of the peak response.

With a damping ratio $\xi = 1\%$, the maximum vertical deflections in proximity to the lost cable are reduced by a maximum of 5% compared to the undamaged structure for the loss of a long cable. For the loss of shorter cables, damping does not have an effect since the peak response occurs directly after the beginning of the vibration. This also applies to the positive bending moments in the bridge girder where damping leads to a maximum reduction of 8% in proximity to the lost cable when the loss of a long cable is being considered. Concerning the cable forces in the cables adjacent to the lost cable, a damping of $\xi = 1\%$ does not reduce the maximum amplitudes. Maximum responses are more strongly reduced at locations which are further away from the lost cable. The responses, however, are small at these locations and are therefore irrelevant to the design.

Thus, a damping has only marginal effects on the state variables of the bridge girder and the cables forces. This is not the case for the bending moments in the pylons. A rather small damping ratio of $\xi = 0.2\%$ leads to an 11% reduction in the negative and a 6% reduction in the positive bending moments at the pylon base. A damping ratio of $\xi = 1\%$ leads to a reduction of 30% and 15%, respectively. These high reductions are due to the fact that the absolute maximum occurs relatively late in the time history.

5 DYNAMIC AMPLIFICATION FACTORS

The structural response of a cable-stayed bridge to the sudden loss of a cable can either be calculated by non-linear dynamic analyses (compare Section 2) or by a quasi-static approach which accounts for the dynamic effects by a dynamic amplification factor (DAF). Therefore, the cable to be considered for failure is removed from the structural model, and the corresponding cable forces are applied to both anchorage nodes of that cable as static load. A static force which is the static cable force multiplied by a dynamic amplification factor is then applied to the anchorage nodes in the opposite direction to the initial cable force. For single-degree-of-freedom systems, this factor is 2.0. This value is also advised by the PTI Recommendations (2001). But in complex systems such as cable-stayed bridges, other values are to be expected. Thus, the upcoming 5th edition of the PTI Recommendations will allow the determination of a dynamic amplification factor in a non-linear dynamic analysis, but stipulates a lower limit of 1.5. Calcula-
tions of dynamic amplification factors for cable bridges can also be found in Hyttinen (1994), Zoli & Woodward (2005) and Park et al. (2007).

In the following, the dynamic amplification factor will be determined for the system described in Section 2, with the cables modeled as equivalent truss elements. The dynamic amplification factor is calculated separately for all state variables $S$ in all structural elements. Therefore, the sudden rupture of a cable is modeled as described in Section 2 and the extreme dynamic responses in time history $S_{dyn}$ are determined. Additionally, the responses to the static removal of a cable $S_{stat}$ are calculated. The dynamic amplification factor is obtained by comparing these responses whereas the responses in the initial state $S_0$ are subtracted, respectively:

$$\text{DAF} = \frac{S_{dyn} - S_0}{S_{stat} - S_0}$$

The results show that a unique dynamic amplification factor cannot be specified. Instead, the value is dependent on the location of the ruptured cable as well as the type and location of the state variable being considered.

For the deflections and the bending moments of the stiffening girder, very different dynamic amplification factors result if the rupture of one cable is considered. Amplification factors at locations further away from the ruptured cable are high, which is shown for the deflections in Figure 3. At these locations static responses are small. While the static removal of a cable mainly causes local deflections and bending moments, the sudden removal of a cable excites natural modes with deflections and moments over the whole girder length. Thus, the mainly excited natural modes are not affine to the static deflection curve. The dynamic responses at locations further away from the ruptured cable are, however, irrelevant when considering all cable loss load cases, because only responses in the vicinity of the ruptured cable are design governing. Concerning the positive vertical deflections, a dynamic amplification factor of between 1.5 and 1.8 results here, depending on the ruptured cable being considered. The amplification factor for the positive bending moments in proximity to the ruptured cable lies between 1.3 and 1.6, while that for the negative bending moments between 1.4 and 2.7. In Figure 4, the envelopes of extreme bending moments from all load cases are shown, together with the corresponding amplification factors.

The dynamic amplification factor for the design governing dynamic axial forces in the stiffening girder at the anchorage points of the long cables is between 1.9 and 2.3. The dynamic amplification factor for the design governing dynamic axial forces at the anchorage points of the short cables is several magnitudes higher than 2.0 because of very small static forces. But here, contrary to the deflections and moments of the stiffening girder, the dynamic forces are not negligible.

![Figure 3](image-url)

(a) Extreme bending moments in stiffening girder in the plane of cable rupture due to permanent loads and loss of second longest cable at bridge center; b) Dynamic amplification factors (DAF)
The dynamic amplification factors for the cable forces in the cables adjacent to the lost cable are between 1.35 and 2.0, depending on the lost cable being considered. The higher values occur when a short cable close to the pylon ruptures. The dynamic forces in the cables adjacent to the lost cable are design-governing for all cable failures, as expected. A high amplification factor at locations further away is therefore irrelevant.

Special attention is necessary for the bending moments in the pylons. The dynamic amplification factor for the bending moments over the whole pylon height and for all cable losses is significantly higher than 2.0. At the pylon base values of about 20 for negative and about 8 for positive moments occur. The static moments are small. However, the dynamic bending moments in the pylon are significant (Fig. 5). Here too, higher modes which are not affine to the static deflection curve are excited. Furthermore, the pylon is not only excited by the impulsive loading of the failing cable, but each of the cables, whose anchorage points are evenly distributed over a length of 26 m at the pylon head, also induces irregular forces which are composed of the redistributed loads from the failed cable plus the inertial forces from the bridge deck. These forces cause a complex structural response which cannot be simplified as described above.

The results show that in the present case, dynamic amplification factors can, at the most, be chosen smaller than 2.0 for the bending moments in the stiffening girder, since over wide parts, the values are smaller. For the design of the cables, a dynamic amplification factor of 2.0 is necessary. For the axial forces in the bridge deck as well as the bending moments in the pylons,
high dynamic forces occur due to the sudden cable loss which cannot be safely accounted for by a quasi-static analysis using amplification factors. In particular, when the static removal causes a decrease in responses while the dynamic removal causes an increase, quasi-static analyses cannot yield correct results. Dynamic analyses seem vital here.

The results shown in this section are for the undamped system under self-weight without live load. However, they are independent of the type of loading. Similar dynamic amplification factors are obtained for a system which is additionally loaded by live load on the main-span or back-spans. As stated in Section 4, a damping only has a small effect on the state variables of the stiffening girder and the cables. Therefore, the impact on reducing the dynamic amplification factor is small. The bending moments in the pylons are significantly reduced even by a small damping. However, the dynamic amplification factors are still much higher than 2.0.

6 COLLAPSE BEHAVIOR

To trace the collapse progression after an initial failure of one or more cables, geometrically and materially non-linear dynamic analyses are necessary. Alternatively, especially for buildings, a quasi-static non-linear analysis has been proposed where the load is incrementally increased to follow the formation of inelastic regions. However, this kind of analysis is only valid if the dynamic response is dominated by a mode shape which is affine to the static deflection curve of the structure without the ruptured cable. This is, as stated in Section 5, not always the case. Thus, dynamic non-linear analyses are conducted here and first results are presented. The procedure is described in Section 2. Concerning the modeling of the cables, taking into account the cable sag and transverse vibrations can lead to slightly lower responses than using simplified modeling (Section 3). In case of the loss of one cable, reasons for this are predominantly the cable’s dynamic properties. For the failure of more cables, the static non-linear behavior becomes important due to higher cable stress variations for which the static stiffness cannot be linearized. Thus, cables are modeled with sag and distributed masses.

In the present case, the failure of a single cable does not lead to a collapse progression. Only local plastifications develop at the anchorage point of the ruptured cable in the affected longitudinal girder and deformations are not significant. Also, the cable tensions remain comparatively small. Live load can be increased by a factor of three or more until the ultimate state is reached. At this state, cables begin to yield and deck deformations strongly increase. Also for the simultaneous failure of two adjacent cables, the bridge remains intact. Deformations are higher and larger plastic sections develop in the bridge girder. The tensions in neighboring cables, however, remain below the yielding stress. If more cables fail, especially short cables, damage will progress and collapse cannot be arrested. Within the range of short cables, high local deformations occur in the bridge girder which are amplified due to high pressure stresses. Upward deflections are accompanied by slackening of cables. At the beginning, the prevailing collapse type is instability-type collapse which may later on be accompanied by a rupture of cables. This is also termed zipper-type collapse. Further investigations are necessary here. Definitions of collapse types can be found in Starossek (2007a).

Generally, robustness has been defined as the insensitivity to local failure. The size of the local failure has to be quantified by the design criteria (Starossek & Wolff 2006). Robustness is, thus, always related to the size of the initial failure. Recommendations for cable-stayed bridges require the analysis for the failure of one single cable; other authors propose the failure of cables within a 10 m range (Starossek 2006a). Furthermore, insensitivity, as the amount of damage the structure has to resist, has to be quantified by the design criteria.

For the investigated bridge, which has a slenderness ratio of 1/230 and moderate cable spacing, the bridge girder is the critical structural element. Depending on the number of lost cables, large sections become inelastic. If this has to be avoided, it only can be stated that the bridge is robust to the loss of one cable. Here, alternate paths can develop. Dead loads and the dynamic loads from the sudden cable rupture are transferred through the bridge girder to adjacent cables. Thereby, only local plastifications develop in the bridge girder. If merely total collapse should be avoided but large inelastic sections are allowed, the bridge can be termed robust regarding the loss of two adjacent cables. The failure of more cables is not possible without serious dam-
age or total collapse. The bridge’s robustness is therefore limited to the failure of one or two adjacent cables depending on the predefined design criteria.

7 CONCLUSIONS

The loss of any one cable was investigated by non-linear dynamic analysis of a three-dimensional model of a cable-stayed bridge. The effects of the cable sag and the transverse cable vibrations on the dynamic structural responses were first determined. A comparison with simplified equivalent modeling of the cables showed that taking into account cable sag and transverse cable vibrations generally leads to smaller demands. Exceptions to this are the bending moments in the pylon at a few section locations. Here, simplified modeling can be unsafe. The effects of damping were also investigated. For the stiffening girder and the cables, the effects are small. For the pylon, a damping of 0.2% of the critical damping already causes a significant reduction of bending moments.

By comparing the dynamic and static structural responses to sudden cable loss, dynamic amplification factors, which can be used in quasi-static analyses, were determined. A unique dynamic amplification factor cannot be specified. The dynamic amplification factor depends on the location of the ruptured cable and on the type and location of the state variable being considered. For the bending moments in the bridge girder, a factor smaller than 2.0 seems possible. A dynamic amplification factor of 2.0 is necessary for the safe design of the cables. Regarding the bending moments in the pylons, significantly larger dynamic amplification factors result. Dynamic time-history analyses are recommended, at least for the critical cable losses which yield the highest responses. Finally, it was shown that the bridge cannot sustain the failure of more than two cables without serious damage. Its robustness is therefore limited to the failure of maximum two adjacent cables.

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REFERENCES

PTI (Post-Tensioning Institute) 2001. Recommendations for stay cable design, testing and installation. 4th edition, Cable-Stayed Bridges Committee. Phoenix: PTI.