Numerical and Experimental Evaluation of Flutter Derivatives by Means of the Forced Vibration Method

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ABSTRACT: A numerical approach using Computational Fluid Dynamics (CFD) for the determination of flutter derivatives for arbitrary bridge deck sections is presented. The results are compared with water tunnel test results. For this study, sectional model tests have been carried out in a water tunnel by means of the forced vibration method, an experimental method that corresponds directly to the analytical method used. Various sections have been tested such as box girder sections, the original Tacoma bridge deck section and a rectangular section with an aspect ratio of 1:8.

KEYWORDS: CFD, forced vibration method, bridge, flutter derivatives, stability, section model tests

1 INTRODUCTION

Motion- or self-induced forces can cause the aeroelastic instability phenomenon called flutter. Because flutter leads to large vibration amplitudes or even to the total collapse of the bridge, flutter stability is an important design criterion for long span bridges.

Today, the proof of flutter stability of bridges is usually based on wind tunnel testing. Section model tests have proved to be a comparatively inexpensive experimental tool. Nevertheless, various numerical methods have been developed in recent years to complement or even replace wind tunnel testing. On one hand the understanding of the flow conditions is improved by easy visualization, on the other hand replacing wind tunnel tests by numerical calculations in the preliminary design stage can cut costs.

The quality of calculations using CFD methods does not only depend on the quality of the solver but to a great extent on the modeling itself. Therefore, the modeling, which includes choosing of the boundary conditions, the degree of grid refinement and time discretisation, is also described here. To replace wind tunnel testing, the numerical method must prove to be reliable and robust. The accuracy of the solution depends on modeling errors, discretisation errors and iteration errors. Errors caused by inaccurate modeling assumptions like boundary conditions and partly also the discretisation error can only be detected by comparison with experiments. Results from simulations with coarse modeling assumptions, like the used two-equation turbulence models, have to be compared to experimental results.

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2 THEORETICAL BACKGROUND

2.1 Motion-induced aerodynamic forces

Considering flutter as a dynamic-aeroelastic stability problem, the motion of the bridge deck at the stability border is assumed to be a sinusoidal motion with constant amplitude. The bridge deck involves two types of deformation: bending and twist. Based on the analytical theory of Theodorsen [1], a modified theory is introduced by Starossek [2], [3]. This theory allows using experimentally or numerically derived flutter derivatives instead of analytically derived ones. For a two-degree-of-freedom system (sectional model), the equation of motion can be written

\[ M\ddot{x} + C\dot{x} + Kx = F_L(t) \]

with \( x(t) = \begin{bmatrix} h(t) \\ \alpha(t) \end{bmatrix} \) and \( F_L(t) = \begin{bmatrix} L(t) \\ M(t) \end{bmatrix} \) (1)

\( M, C \) and \( K \) are the diagonal mass, damping and stiffness matrices, \( x(t) \) is the displacement vector and \( F_L(t) \) is the vector of self-excited aerodynamic lift and moment (Fig. 1).

Following the complex number approach of Starossek [2], [3], the aerodynamic forces can be expressed in the following linearized form:

\[ L(t) = \pi \rho k^2 u_a^2 [c_{hh} h(t) + b c_{ha} \alpha(t)] \]  

\[ M(t) = \pi \rho k^2 u_a^2 [b c_{ah} h(t) + b^2 c_{aa} \alpha(t)] \]  

(2)

(3)

The \( c_{mn} = c_{mn}' + i c_{mn}'' \) are the dimensionless complex force coefficients or derivatives, where \( c_{mn}', \) and \( c_{mn}'' \) are real numbers and \( i \) is the imaginary unit. Furthermore, \( k = \omega b/u_a \) is the reduced frequency, \( b \) is the half width of the bridge deck, \( u_a \) is the velocity of the oncoming flow, \( \omega \) is the circular frequency of structural motion and \( \rho \) is the air density. The \( c_{mn} \) are functions of \( k \). The first index describes the direction of the force; the second index indicates the motion causing the force. This formulation is based on the complex description of flutter derivatives by Kloeppel et al. [4] and is preferred by the authors. Four complex coefficient functions thus describe the relationship between motion and self-excited wind forces.
Alternatively, following the real number approach of Scanlan in its latest form [5], the aerodynamic forces can be expressed as

$$L(t) = \frac{1}{2} \rho u_\infty^2 B \left[ KH_1^*(K) \frac{\dot{h}(t)}{u_\infty} + KH_2^*(K) \frac{\dot{B}\dot{\alpha}(t)}{u_\infty} + K^2 H_3^*(K) \alpha(t) + K^2 H_4^*(K) \frac{h(t)}{B} \right]$$

(4)

$$M(t) = \frac{1}{2} \rho u_\infty^2 B^2 \left[ KA_1^*(K) \frac{\dot{h}(t)}{u_\infty} + KA_2^*(K) \frac{\dot{B}\dot{\alpha}(t)}{u_\infty} + K^2 A_3^*(K) \alpha(t) + K^2 A_4^*(K) \frac{h(t)}{B} \right]$$

(5)

where the reduced frequency $K$ is defined as $K = B \omega / u_\infty$ and $B$ is the deck width. The coefficients $H_i^*$ and $A_i^*$ are the flutter derivatives based on Scanlan’s approach. The relationship between these two formulations is as follows

$$c_{hh}' = \frac{2}{\pi} H_4^*, \quad c_{hh}'' = \frac{2}{\pi} H_1^*, \quad c_{ah}' = \frac{4}{\pi} A_4^*, \quad c_{ah}'' = \frac{4}{\pi} A_1^*,$$

(6)

$$c_{ha}' = \frac{4}{\pi} H_3^*, \quad c_{ha}'' = \frac{4}{\pi} H_2^*, \quad c_{aa}' = \frac{8}{\pi} A_3^*, \quad c_{aa}'' = \frac{8}{\pi} A_2^*,$$

(7)

Starossek [6] discussed and compared both methods of analysis. The real part of the $c_{mn}$ represents the component of the aerodynamic force that goes along with structural displacement, the imaginary part is related to velocity (Fig. 1). One advantage of complex notation is the easy identification of the derivatives by the indices. Another advantage is the more compact and natural representation of aerodynamic forces and of the ensuing eigenvalue problem.

All results are plotted over the dimensionless reduced wind speed $U_{red} = u_\infty / B f$, where $f$ denotes the frequency of structural motion. The relationship between $k$ and $U_{red}$ is

$$U_{red} = \frac{\pi}{k}$$

(8)

2.2 The forced vibration method

In both, the numerical and the experimental simulations, the sectional model of the bridge deck is forced to move either vertically to the flow direction [$h = h \sin(\omega t)$] or rotationally [$\alpha = \alpha \sin(\omega t)$] in a sinusoidal motion with constant amplitude. Thus always two sets of simulations are carried out. The aerodynamic forces are measured experimentally or calculated by means of CFD.
The coefficients $c_{hh}$, $c''_{hh}$, $c'_{oh}$ and $c''_{oh}$ are extracted from the aerodynamic forces caused by forced vertical motion, the coefficients $c'_{ha}$, $c''_{ha}$, $c'_{aa}$ and $c''_{aa}$ are extracted from the aerodynamic forces caused by forced rotational motion. The forces are extracted from the time domain data by means of spectral analysis.

2.3 Modeling of Fluid-Structure Interaction

Fluid and Structure are modeled separately. This leads to a particularly simple procedure when the structural motion, at the critical wind speed (stability border), is assumed to be a constant-amplitude sinusoidal motion. The flutter derivatives determined either experimentally or numerically are introduced into the structural analysis and the critical wind speed is calculated. This can be done either by a generalized two degree of freedom (2-DOF) algorithm or through a more precise FEM model. Starossek [2], [3] describes both kinds of analysis.

H-Sections like the tested Tacoma Narrows Section (Fig. 2: TC) show a significantly different aeroelastic behavior compared to box girder sections. The response of such sections is almost pure rotational motion. This can be described by a simplified mathematical model with one degree of freedom (1-DOF), which leads to the following stability criterion:

$$c''_{aa} = ga\left(\frac{I_a}{\pi \rho b^4} + c'_{aa}\right)$$

(9)

where $I_a$ is the mass moment of inertia and $g_a = 2\xi_a$ is the structural damping coefficient, where $\xi_a$ denotes the damping-ratio-to-critical (Starossek [2], [7]).

3 EXPERIMENTAL SET-UP

To investigate the reliability of the numerical method, various sections were examined experimentally (Fig. 2). Experimentally evaluated flutter derivatives $c'_{aa}$ and $c''_{aa}$ for selected sections are shown in Figure 3. The tests have been carried out in the water tunnel of the IAG, University of Stuttgart. The completely submerged bridge deck section model is cantilevered from the balance system vertically. Controlled by a balance system the bridge deck section model is forced to move in harmonic oscillations. The experimental set-up is illustrated by Bergmann [8].

The turbulence intensity of the water tunnel is 0.8 %. The influence of the turbulence intensity was not further investigated experimentally.
The velocity range of the water tunnel is 0.3 to 2.5 m/s. With a constant bridge width B of 200 mm, the Reynolds number varies between $1 \times 10^5$ and $2.5 \times 10^5$. In this range, the influence of the Reynolds number has proved to be small according to the experimental results.

A certain range of motion amplitudes has been tested as well. Because the motions are assumed to be small in theory, the amplitudes were restricted to a rotational motion of $8^\circ$ and a vertical motion of 4% of B. Only the TC section shows a significant variation in behavior between $2^\circ$ and $8^\circ$ degree amplitudes. Using (9) and assuming $g_o$ to be zero, the smaller amplitude motion leads to a critical reduced wind speed of 3.92. This value is approximately 25% smaller than the corresponding value resulting from the derivatives determined from the larger amplitude motion.

4 NUMERICAL SET-UP

The numerical simulations have been carried out by the CFD-Program COMET, which is based on the Finite Volume Method. The solver is an Incomplete Cholesky Preconditioned Conjugate Gradient Solver. The flow is modelled to be two-dimensional and either laminar or turbulent. To model the turbulence, two-equation-models, such as RNG k-ε model and Wilcox k-ω model, are used. Assuming the velocity being uniform in the far field, the whole grid is moved in the prescribed motion for each degree of freedom separately. To model the rigid body motion of the grid for laminar flow, a surface velocity $v_s$ is added to the continuity equation

$$\frac{d}{dt} \int_V \rho \, dV + \int_S \rho (v - v_s) \cdot ds = 0$$

and to the convection term of the momentum equation

$$\frac{d}{dt} \int_V \rho v \, dV + \int_S \rho (v - v_s) \cdot ds = \int_S T \cdot ds + \int_V f_s \, dV$$

If turbulence is modeled, the additional velocity component has also to be incorporated into the convection terms of the transport equations of the kinetic energy k and its dissipation rate $\varepsilon$. Equations (10) and (11) are also extended according to Reynolds averaging in the case of turbulence modeling, but they are not affected by the velocity $v_s$.

The results of turbulent and laminar modeling do not differ significantly because the flow in the water tunnel has a very small turbulence intensity. However, the additional diffusion caused by the additional turbulent viscosity stabilizes the numerical solution. Numerical oscillations that can disturb the time dependent solution are damped that way. This leads to smoother solutions. All numerical results presented in this paper are obtained by means of turbulence modeling.

The grid has been significantly refined near the structure surface along the estimated depth of the boundary layer (Fig. 4). Near edges the grid has also been more strongly refined. After a preliminary run, the grid has been refined again guided by the local velocity error. The far field is modeled with a coarse grid, because the velocity is assumed to be uniform there. A central differencing scheme is chosen according to the fine grid.

The time step has to meet two criteria: a sufficiently fine discretisation of the amplitude length $T = 2\pi/\omega$ (> 1000 time steps per circle) and a Courant number of less than 1. An Euler implicit time integration scheme is used.

The inflow boundary condition at the left, top and bottom boundary is a uniform velocity $u_\infty$ with a turbulence intensity of 1% and a turbulence length of 0.001 m. Figure 4 shows the position of the boundaries. The definition of an inlet boundary on top and bottom of the model is necessary to provide a uniform velocity $u_\infty$ in the far field when the grid is rotated.
A boundary velocity equal to the first derivative of the motion is defined at the structure surface, where the velocity parallel to the surface is assumed to be zero. The forces on the surface of the structure are determined by integrating the boundary element forces.

5 COMPARISON OF EXPERIMENTAL AND NUMERICAL RESULTS

Only results with the same Reynolds number in experiment and numerical simulation are compared. The results for section S are shown in Figure 5. Numerical and experimental values are in good agreement. For sake of comparison, Theodorsen’s [1] analytical results for a thin flat plate are also shown. The flutter derivatives of the plate-like sections GB and M, not shown here, are also in good agreement to each other. For the results for the GB section, see Thiesemann et. al [9]. For all these sections, the flutter derivatives are relatively close to the derivatives for a flat plate according to Theodorsen [1]. Sections with this characteristic behavior are denoted as plate-like sections.

As a quantitative measure for the quality of the numerically evaluated derivatives, the critical wind speeds, for various example systems, based on either numerically evaluated or measured flutter derivatives have been compared. For sections S, GB, P and M the differences of critical wind speeds are small (i.e., in the range of 5%).
Results for two sections prone to rotational flutter are shown in Figure 6. The numerically and experimentally evaluated derivatives are in good agreement in case of the TC section — approximately 10% difference of predicted critical wind speeds when using (9). In the case of the R section, however, numerically and experimentally determined derivatives differ significantly. On the other hand, the numerical results for section R show good agreement with the wind tunnel test results of Hortmanns [10]. This discrepancy has to be further investigated both ways, experimentally and numerically.

The results for the twin box section G and the plate section P show that the respective tendencies in aerodynamic behaviour of these sections are well captured (Fig. 7). The results of section P, a 1:25 rectangular section, indicate a plate-like behaviour.

6 CONCLUSIONS

The numerical results show a reasonable agreement with the data determined by sectional model tests. This has been achieved with a relatively simple and computationally inexpensive method for modeling turbulence ($k-\varepsilon$ model). Still, numerical simulations should be used accompanied by experiments, before the latter can be replaced completely.
On the other hand, experimental results may also show discrepancies, as in the case of the TC and R sections. Numerical simulations are thus a possibility to conversely check experimental results. As already common practice for stationary wind load coefficients, wind tunnel testing should be complemented by numerically evaluated data to improve the degree of certainty.

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