Measures of Structural Robustness – Requirements & Applications

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INTRODUCTION
The investigation of structures concerning their susceptibility to progressive disproportionate collapse is attracting increasing attention. At present, there is neither a uniform theory of progressive collapse and collapse resistance nor any agreement on terms and nomenclature. Codes and other relevant literature require a reduction in the susceptibility to progressive collapse. However, mainly qualitative and hardly any quantitative recommendations are provided.

Collapse resistance can be influenced in various ways. One possibility is through the structural robustness. In a robust structure, no damage disproportionate to the initial failure will occur. To examine a structure in terms of its robustness, a quantitative description of the robustness by means of a measure would be useful.

In this work, basics for the development of measures to quantify structural robustness are formulated. Essential definitions are suggested and the requirements for measures of robustness as well as possible applications are discussed. A selection of publications on the quantification of robustness, or related characteristics such as vulnerability, are presented and discussed regarding the proposed requirements and applications.

PROGRESSIVE COLLAPSE
Progressive collapse is characterised by a pronounced disproportion between the magnitude of a triggering local event and the resulting widespread collapse of large parts or the entire structure [Starossek, 2007b]. Sometimes the term disproportionate collapse is also used. Progressive collapse can be produced by different mechanisms which depend on the type and form of a structure and its orientation in space, as well as the type, location and magnitude of the triggering event [Starossek, 2007a].

The probability of a progressive collapse \( P(F) \) as a result of an abnormal event can be represented as a chain of partial probabilities [Ellingwood and Dusenberry, 2005]:

\[
P(F) = P(F \mid DH) \cdot P(D \mid H) \cdot P(H)
\]  

(1)

In this equation, \( P(H) \) denotes the probability of an abnormal event that threatens the structure or more generally a hazard \( H \) for the structure; \( P(D\mid H) \) is the probability of local damage \( D \) as a result of the event \( H \); and \( P(F\mid DH) \) denotes the probability of the failure \( F \) of the structure as a result of local damage \( D \) by \( H \).

The probability of a progressive collapse must be limited to a generally accepted value. The three partial probabilities described above can be modified through the associated strategies influencing abnormal events, influencing individual local element behaviour and influencing...
global system behaviour. These possibilities are described in more detail elsewhere [Starossek, 2007b] [Ellingwood and Dusenberry, 2005] [EN 1991-1-7, 2006].

Influencing abnormal events is generally not within the control of a structural engineer but the probabilities $P(D|H)$ and $P(F|DH)$ are. The common standards usually describe the safety of a structure as a function of the safety of all elements against local failure, and therefore only take into account the probability $P(D|H)$. The reaction of a structure to the possible occurrence of a local failure – expressed by the probability $P(F|DH)$ – is seldom investigated. Only important buildings are investigated regularly for this case.

In the literature, a large number of terms with various meanings are used in connection with progressive collapse. In this paper, the terms robustness and collapse resistance will be used according to the definitions by [Starossek, 2006], which are also used in [SEI-PCSGC, 2007]. Robustness is associated with the probability $P(F|DH)$, and collapse resistance with the probability $P(F)$ (see Figure 1). Other terms may also be used; however, it is fundamental that a clear distinction is made between the properties outlined below.

**ROBUSTNESS**

The term robustness is defined as insensitivity of a structure to local failure, where insensitivity and local failure must be quantified by so-called design objectives [Starossek 2006] [Starossek, 2007b]. It is a property of the structure alone and independent of the possible causes and probabilities of the initial local failure.

**COLLAPSE RESISTANCE**

The term collapse resistance – the short form for resistance against progressive collapse – is defined as insensitivity of a structure to accidental circumstances, which comprise unforeseeable or low-probability events [Starossek 2006]. Accidental circumstances themselves and their possible effects on the structure must be defined and quantified by design objectives. Collapse resistance is a property that is influenced by both structural features as well as possible causes of initial failure.

**FURTHER TERMS**

The term robustness is often used synonymously with, as well as in contrast to, a number of other terms. In the context of progressive collapse, frequently used terms to describe structural characteristics are briefly presented alphabetically below:
- **Continuity**: Continuity refers to the continuous connection of components as well as the continuous reinforcement of concrete components. Integrity, redundancy and/or local resistance can be improved and special load-carrying mechanisms enabled by continuity. Continuity is thus an element of robustness. However, its benefit is a matter of controversy [Starossek, 2006].

- **Damage tolerance**: The term damage tolerance used in [Lind, 1995] and [Frangopol et al., 1991] is compatible with the term robustness defined in this paper. In [Bontempi et al., 2007], the damage tolerance is defined in narrower terms and refers to the ability of a structure to resist a continuous local deterioration due to corrosion or similar.

- **Ductility**: Ductility is the ability of a component or structural system to withstand large plastic deformations. Ductility has a large influence on progressive collapse and is often listed as a factor which increases the robustness of a structure. Nevertheless, measures of ductility are not suitable for expressing the robustness of a structure.

- **Integrity**: The term structural integrity is mainly used in North American standards [ASCE 7, 1998] [ACI 318, 2002], often in relation with prescriptive requirements (such as requirements for continuity, ductility and redundancy). Integrity refers to the condition of a structural system and implies that the structure and its components remain intact over the intended lifetime of the structure. Enhanced integrity may mitigate progressive collapse.

- **Redundancy**: Structural redundancy refers to the multiple availability of load-carrying components or multiple load paths which can bear additional loads in the event of a failure. If one or more components fail, the remaining structure is able to redistribute the loads and thus prevent a failure of the entire structure. Redundancy depends on the geometry of the structure and the properties of the individual load-carrying elements [Frangopol and Curley, 1987]. It is not synonymous with static indeterminacy. Redundancy is mentioned several times as an important factor in the design of robust structures and hence the prevention of progressive collapse [EN 1991-1-7, 2006]. In that context, redundancy refers in particular to the design method alternative load paths.

- **Vulnerability**: The term vulnerability is used in [Lind, 1995], but particularly in [Agarwal et al., 2003] and [Agarwal and Blockley, 2007]. Vulnerability describes the sensitivity of a structure to damage events. A structure is vulnerable if small damages lead to disproportionate consequences. Vulnerability is antagonistic to robustness and it is a property of the structural system.

**AREAS OF APPLICATION**

The quantification of robustness and collapse resistance opens a number of different areas of application. However, first the two questions *What is a measure for robustness or collapse resistance?* and *What is the purpose of quantification?* must be answered. The measure should quantify the robustness or collapse resistance of a structure with one single value. It should express how and to what extent design objectives are influenced by abnormal events. With this, a number of areas of application exist [Haberland, 2007] [Starossek, 2007b]:

- **Optimisation**: If the robustness or collapse resistance of a structure can be quantified unequivocally, then knowing this value allows it to be optimised.

- **Regulation**: In order to regulate robustness or collapse resistance, quantification is required. Requirements can be defined in standards and codes generally or according to the type of structure and depending on the significance and exposure of the building.
• **Evaluation:** By using quantitative measures, the significance of specific damage scenarios can be monitored and critical elements identified. Furthermore, different design variants can be compared and sorted in certain manner.

• **Design aid:** The measures should be a decision-facilitating aid for the design of structures. Based on the findings for robustness or collapse resistance, the question of whether they are sufficient and the use of conventional structural design methods suffices, or whether additional investigations and measures for progressive collapse are required – including preferable design strategies – could be decided [Starossek, 2006].

• **System partial safety factors:** It is conceivable that future generations of standards based on the partial safety factor concept will be supplemented by a system partial safety factor on the resistance side, which takes into account the response of the structure to initial failure – and hence the measure of robustness.

**Requirements**

The validity and usefulness of measures of robustness and collapse resistance is linked to a series of general requirements [Haberland, 2007] [Starossek, 2007b] [Lind, 1995]:

• **Expressiveness:** The measure should express all aspects of robustness and collapse resistance but no other aspects. It should allow for a clear differentiation between robust and non-robust or collapse-resistant and collapse-susceptible structures. For this, compliance with given design objectives is to be checked.

• **Objectivity:** The measure should be independent of user's decisions. The result for the measure should be reproducible under the same conditions.

• **Simplicity:** In the interest of objectivity and generality as well as for promoting acceptance, the definition of the measure should be as simple as possible.

• **Calculability:** It should be possible to derive the measure from the attributes or the behaviour of the structure. All necessary input parameters must be quantifiable. The numerical calculation of the measure should not require excessive effort and should be sufficiently accurate.

• **Generality:** The measure should be applicable to arbitrary structures.

These requirements are partly in conflict with each other, so it may not be possible to fulfil them all to the same level at the same time. An investigation of the approaches proposed up to now indicates that while it is possible to achieve strong expressiveness this is done at the cost of
calculability. Hence the above requirements may have to be limited. This seems plausible and justifiable at least with regard to the generality requirement. As discussed in [Starossek, 2006], different types of structures have a tendency for particular significant mechanisms of collapse. Measures that are specific to types of collapse appear to be favourable for a realistic description of structural behaviour.

QUANTIFICATION TERMS

For a description of properties, there are several qualitatively and quantitatively oriented terms. A selection of the most common terms is presented below and their application is put into context:

- **Indicator**: The term indicator is used in many disciplines. In this paper, the term is used to describe the meeting of or departure from certain conditions and circumstances. An indicator is a qualitative description of a property.
- **Measure**: A measure is used for the quantitative description of a property.
- **Index**: An index is a ratio for the change of values of a quantity. It is used for the quantitative description of a property at a fixed scale.

There is a relation between the above defined terms. An indicator is the most general form used to describe a property. Each measure can also be regarded as an indicator, and an index is a special form of a measure.

PRESENT APPROACHES

Various approaches for the quantification of robustness, or related characteristics such as vulnerability, have been published. Below, a selection is presented and subsequently evaluated regarding the applications and requirements identified above.

Lind [1995]

Damage tolerance is a desirable characteristic of a system and is defined as the insensitivity of the system to local damage. Vulnerability is a complementary concept. A probabilistic measure based on the comparison of the probability of failure of a damaged to an undamaged structural state is proposed. It is a relative measure to study and compare the structural effects of an assumed damage.

Wisniewski et al. [2006]

A method to assess the safety of railway bridges based on the investigation of the load-carrying capacity of the system is presented. Robustness is one aspect here and is considered as the ability of structural systems to continue carrying loads after failure of one of their elements. The quantification of bridge safety is done by means of so-called redundancy ratios which compare the load-carrying capacity in limit states with the design load-carrying capacity. Probabilistic calibration coefficients which depend on the respective limit states are also included.

Maes et al. [2006]

Three different measures for the quantification of robustness are proposed and discussed. Robustness is defined as the ability of a structure to resist damage as a result of extreme
disturbance. Two measures are defined by the relationship between the original and damaged structural states. The system load-carrying capacity as well as the probability of failure of the structure is used as a comparative value. A third measure is based on a detailed risk-consequence analysis.

Smith [2006]

Based on the hypothesis of an analogy between progressive collapse and the fast fracture of metals, an energy-based approach to assess the robustness of structures is proposed. The minimum required energy to damage a sufficient number of elements, leading to the collapse of the structure, is defined as an indicator of structural robustness. Different structural arrangements can be compared in this way and highly critical failure scenarios can be identified.

Starossek [2007b]

Three approaches to the quantification of structural robustness based on damage, energy and the stiffness matrix are introduced. Robustness is the insensitivity to local damage. Closely related to this definition, a measure is suggested as the complement of the referenced damage progression (damage progression due to local initial damage related to a limit value specified by design objectives). The extension of this measure defined by an integral, which considers the damage progression after varying degrees of initial damages, offers a more comprehensive insight into the structural behaviour. The formulation based on the stiffness matrix describes the structural robustness using a ratio of the system structural stiffness (determinant of the system stiffness matrix) of the damaged to that of the undamaged structural state. The comparison of the energy released by an initial failure with the energy required for a collapse progression also leads to a measure.

Harte et al. [2007]

A two-part measure to quantify the robustness of structures is presented. Robustness describes the property of structures to resist an unforeseen or extraordinary event according to specified design objectives. The measure consists of a reserve load-capacity factor of the undamaged structure and a factor to describe the evolution of damage due to load increase. For the latter, damage indicators based on eigenvalues of the system tangent stiffness matrix are used.

Agarwal et al. [2003]

To investigate a structure regarding disproportionate collapse, a so-called theory of structural vulnerability was developed. The theory examines the form and connection of the structure and is independent of possible events as well as their probability. A hierarchical model of the structure is created based on form and connection. This model is used to identify inherent weaknesses and possible failure scenarios. The importance of a failure scenario is described by a vulnerability index. This measures the vulnerability of a structure by the relationship between the structural consequences to the related associated damage demand of the failure scenario. Both dimensions are based on the system stiffness matrix. Failure scenarios where the initial damage and the consequences are disproportionate can be subsequently examined for the likelihood of the failure under specific events and loads. The vulnerability theory can complement the usual security, reliability and risk investigations.
A measure of the robustness is introduced based on probabilistic risk assessment. Robustness is defined by the proportionality of consequences of a structural damage to the cause. The approach divides consequences into direct consequences associated with the damage of a local element and indirect consequences associated with the subsequent system failure. The measure results from the comparison of direct risks with the total risk (sum of the direct and indirect risks). Risk is a product of consequence and probability of occurrence. The fewer indirect risks involved in the total risk, the more robust a structure is. The probabilistic formulation allows the robustness of different systems to be compared.

**Comparison and Evaluation**

A variety of approaches to the formulation of measures for robustness or related characteristics were published. In principle, it is possible to distinguish between measures based on structural behaviour and those based on structural attributes, with the first group clearly prevailing. The measures based on the structural behaviour – also called performance-based measures – can be further divided into deterministic and probabilistic measures. In the former case, the system load-carrying capacity [Maes et al., 2006] [Wisniewski et al., 2006] [Harte et al., 2007], the extent of damage progression [Starossek, 2007b] [Haberland, 2007] or energy [Smith, 2006], [Starossek, 2007b] are used. The latter looks at the probability of failure [Lind, 1995] [Maes et al., 2006] or at the risk [Baker et al., in press]. Measures based on structural attributes quantify the system stiffness [Starossek, 2007b] [Haberland, 2007] or are based on topological examinations of the structure [Agarwal et al., 2003]. This classification is shown in Figure 3. Further distinction can be made between measures based on the assumption of an initial damage and the investigation of the resulting effects and those based on the identification of a collapse sequence.

Measures to investigate an initial damage are often based on the comparison of two structural conditions. Various methods of comparison are used: (1) Measures which compare the damaged and undamaged state – regardless of whether probabilistic, deterministic or otherwise – often do not seem to be highly expressive [Haberland, 2007]. The compliance with given design objectives is not verified so that a distinction between robust and non-robust structures is not immediately possible. Such measures rather indicate the importance of the damaged component and not the structural robustness. (2) By comparing the damaged state of the structure with a reference value defined by design objectives, a more expressive measure can be derived. (3) Comparisons based on the undamaged state of the structure and a reference value hardly represent an expressive measure. For the special case where the load-carrying capacity is used as the comparative value, these three alternatives could be named residual strength, reserve residual strength or reserve strength factor respectively. They were proposed as so-called measures of redundancy two decades ago [e.g., Frangopol and Curley, 1987].

A detailed comparison [Haberland, 2007] shows that the approaches suggested up to now fulfil the presented requirements to varying degrees, but none has proven to be clearly superior. However, the requirements are partly in conflict with each other, so it may not be possible to fulfil them all to the same level at the same time. Hence the requirements may have to be limited. This seems plausible at least with regard to the generality requirement. Different types of structures have a tendency for certain significant mechanisms of collapse. The development of an all-encompassing measure seems difficult. For practical applications, specialised measures that are specific to types of collapse are necessary [Starossek, 2007a].
Measures based on the structural behaviour generally require dynamic calculations, taking into account geometric and material non-linearity; separation and impact processes are also possible. Depending on the type of structure and the controlling collapse mechanism, the necessary calculations here are either too complicated or require too much effort for practical purposes so that simplified analysis methods are used. While nonlinear structural behaviour is usually considered in the quantification proposals surveyed above, dynamic effects and in particular impact loads are neglected. These, however, have a significant influence on the progression of a collapse, at least for pancake-type collapses. Another problem is the specification of a realistic load on the structure. The benefit of a measure decreases as the complexity of its calculation increases.

These measures often appear expressive but cannot be realistically calculated because of the extensive structural analyses required. Despite continually growing numerical calculation capacities, a realistic analysis of structural behaviour as a result of an assumed initial damage will require great effort. Under this premise, the development of simple and calculable measures based on structural attributes for the prediction of structural behaviour appears to be desirable for practical applications. On the other hand, the expressiveness of such measures proposed up to now is not yet adequate.

Probabilistic measures are theoretically universal; however, their realistic assessment, if at all possible, requires great effort. They are generally too complex to be useful in typical design situations [Canisius et al., 2007]. Nevertheless, they can be used as a tool in evaluating the performance of simplified robustness measures.

**CONCLUSIONS**

While collapse resistance is a characteristic which is influenced by both structural features as well as possible accidental circumstances, robustness is a characteristic only of the structure. To examine a structure in terms of its robustness, a quantitative description of the robustness would be useful. The basics for the development of measures to quantify structural robustness were introduced. Essential definitions were suggested and the assessment, optimisation and regulation of structural robustness as applications of measures of robustness were identified. To achieve these tasks, the measure must be expressive, objective, simple, calculable and generally applicable. These requirements are partly in conflict with each other, so it may not be possible to fulfil them all at the same time.
A selection of the various published approaches to quantify robustness, or related characteristics such as vulnerability, was presented. Most of the approaches lead to performance-based measures defined by the change in structural behaviour due to an assumed initial local failure and quantified by probabilistic or deterministic comparative values. Some measures require the determination of collapse scenarios, which is numerically challenging. Other approaches are based on structural attributes.

It was shown that the suggested approaches fulfil the requirements to different degrees. No approach has shown itself to be clearly superior. Measures based on structural behaviour often seem to be expressive but can hardly be calculated because of the extensive structural analyses that are required. On the other hand, measures that are based on structural attributes are usually easier to calculate while their expressiveness is not yet adequate. Different types of structure have a tendency for particular types of collapse, which can be described favourably by specifically defined measures.

Despite continually growing numerical calculation capacities, a realistic analysis of structural behaviour as a result of an assumed initial damage will require great effort. Under this premise, the development of expressive measures based on structural attributes for the prediction of structural behaviour appears to be desirable for practical applications.

REFERENCES

ACI 318 [2002], "Building code requirements for structural concrete (ACI318-02) and commentary (ACI318R-02)", American Concrete Institute (ACI), 2002.


ASCE 7 [1998], "Minimum design loads for buildings and other structures", American Society of Civil Engineers (ASCE), 1998.


Haberland, M. [2007], "Progressiver Kollaps und Robustheit (Progressive collapse and robustness)". Hamburg University of Technology, Structural Analysis and Steel Structures Institute, Diploma thesis, 2007; www.sh.tu-harburg.de. – (in German)


SEI Progressive Collapse Standards and Guidance Committee (PCSGC) [2007], "ASCE/SEI Pre-Standard: Mitigation of disproportionate collapse in building structures. Annotated outline. May 16, 2007".


