Problems and Opportunities of Nonlinear RC Frame Analysis

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Summary

A safe and economic realisation of tall buildings and other slender structures calls for accurate structural analysis. Several current analysis programs allow for a materially and geometrically nonlinear analysis of reinforced concrete frames, either by frame or three-dimensional elements. For the evaluation of these programs the authors suggest a benchmark. The benchmark comprises seven testings and reveals a number of distinct differences between numerical results and experimental data. Most problems arise in the computation of spatial structures. For frame elements, no generally accepted cross-sectional model exists taking into account material nonlinearity for all six internal forces. To overcome these problems a hybrid model for arbitrary cross sections is suggested. The new approach is based on a notional division of the cross section into two areas. Where stirrups and longitudinal bars form a reinforcement net, membrane elements are used. In the remaining area, stresses are assumed to be uniaxial. The presented model allows for a fully nonlinear analysis including interaction between all six internal forces. This provides the opportunity for a unified design procedure. Many current concrete codes already allow a nonlinear determination of the internal forces. However, the limit states are checked separately for biaxial bending and normal force, for transverse forces, and for torsion at cross-sectional level. Interactions are only empirically considered. In contrast, the proposed unified concept is based on the limitation of the principle concrete strains and the reinforcement strain produced by all internal forces. The design procedure is reduced to a single nonlinear analysis of the structure.

Keywords: reinforced concrete, frame, nonlinear analysis, benchmark, cross-sectional analysis, unified design procedure

1. Introduction

Nonlinear analysis of reinforced concrete structures used to be limited to research projects. Current software also provides practicing engineers with nonlinear analysis tools. The new generation of concrete codes further propagates nonlinear analyses. Due to the increasing application of nonlinear analysis programs their reliability has to be ensured. Numerical computations of existing experiments often give the impression of perfect accuracy. However, for the design of new structures not a recomputation but a reliable prediction is necessary. Therefore a series of benchmark problems for reinforced concrete frames is suggested and run with four programs in section 2. The frames are modeled by both three-dimensional and frame elements. For all tested programs some of the numerical results deviate substantially from the experimental data. Regarding frame elements, the study reveals the lack of a cross-sectional model capturing the nonlinearities of all six internal forces. Therefore, a brief description of a new hybrid approach is provided in section 3. Embedding the new model into a nonlinear structural analysis program facilitates a complete interaction between system and cross-sectional level [1]. The model takes into account the nonlinearity and interaction of all six internal forces and provides the corresponding stiffnesses for a fully nonlinear analysis [2]. In current codes and software, the interaction is usually restricted to biaxial bending and axial force. For instance, the influence of shear and torsion on the bending stiffness is neglected or only empirically considered. Including shear and torsional interactions into the analysis provides the opportunity for a unified design procedure, which is suggested in section 4.
2. Benchmark for nonlinear RC frame analysis

2.1 Intention and procedure

Reinforced concrete structures show a complex nonlinear behavior. Some analysis programs promise to capture nonlinear effects, but it is difficult to assess the performance of these programs. As exact analytical reference values do not exist, a benchmark for reinforced concrete analysis is dependent on experimental data. Previous benchmarks for special specimens were run for example in [3]. In contrast, the proposed benchmark concentrates on the analysis of members of practical size and composition. So far the benchmark is limited to frames. It consists of seven types of experiments. The first four investigate the behavior of frame elements separately for normal force, shear force, bending moment and torque. For biaxial bending as well as biaxial shear force the load can always be reduced to a one-directional load by regarding a rotated cross section. To investigate geometric nonlinearity a column under eccentric axial compressive load is included. Important effects of the nonlinear analysis are the redistribution of internal forces in statically indeterminate structures and the interaction of the internal forces. Therefore a two-hinged frame and a grid are analyzed. For each system type several examples are investigated. The fundamental behaviors prove to be quite similar, so only an assortment of seven experiments is presented. Out of the wide range of programs four — in the following referred to as A, B, C, and D — are selected for the benchmark. Currently the benchmark series is being extended. All tested programs provide special material or element formulations for concrete. In A and B the specimens are modeled by three-dimensional finite elements with embedded reinforcement. In C and D frame elements are used. Program A is based on an ideal elastic-brittle constitutive model with three-dimensional failure criteria. The elements of B and C both apply the same elasto-plastic damage model. In D a nonlinear transfer matrix method is implemented. All four models are based on a smeared cracked approach to represent the discontinuous macrocracks. Material parameters which are not directly available are derived following [4], otherwise standard model settings are adopted.

2.2 Experiments

2.2.1 Axial force

The first part of the benchmark addresses the behavior of plain concrete under a uniform uniaxial stress state. The concrete is loaded and unloaded both under tension and compression. Important phenomena are cracking in tension and softening in compression, plastic strain, crack closure and reopening, and the stiffness reduction under cyclic loading. A description of the effects can be found for instance in [5]. For reinforced concrete the behavior under tension is particularly interesting. Smeared crack models consider the tension stiffening effect by a modification of the constitutive laws of either concrete or reinforcement.

Fig. 1 shows the results of the four numerical analyses. Due to its simplified constitutive law program A results in a poor approximation. Between the ultimate tension and compression stresses a linear-elastic behavior is assumed. Beyond this range the stress drops to zero and does not recover.
Curves B and C are congruent and all of the above mentioned phenomena can be identified. The tension curve D shows a linear-elastic brittle behavior. Under compression the nonlinear curve ends with a plateau at the ultimate stress level. During unloading plastic strains are not considered.

2.2.2 Bending moment

The behavior under bending is analyzed on a four-point bending beam. The advantage over a three-point bending beam [3] is the larger undisturbed failure region. Thus, the influence of local hyper-strength, and the problems of interaction with shear-force and the local stress state under the load, are avoided. The numerical analyses are run for beam E5/1 [6]. An interesting aspect is the elongation of the beam under pure bending caused by cracking.

Due to the abrupt failure of concrete under tension A underestimates the stiffness under small loads (Fig. 2). Since the post-failure compression stress is neglected as well, the computed ultimate load of the beam is too low. When the maximum compression stress is reached, the reinforcement immediately starts to yield. Curve B lies below the experimental one and fails to adequately approximate the bearing capacity. The computation of C shows good accuracy but already ends at 50% of the ultimate experimental load. Program D overestimates the stiffness but estimates the bearing capacity quite well.

![Fig. 3: Shear force](image1)

![Fig. 4: Torque](image2)

2.2.3 Shear force

The modeling of the shear behavior is analyzed using the same system as for bending moments but without stirrups. The shear failure develops in the exterior segments between load and support. An interaction with the bending moment is unavoidable. For this reason the shear failure occurs in the compression zone directly next to the loading point. The numerical and experimental curves of beam E6 [6] are plotted in Fig. 3.

Curve A shows a close match, but the stiffness reduction before failure is not captured. The results of B closely follow the plot of the experimental data and reproduce the ultimate load with good accuracy. Curves C and D are nearly the same as in Fig. 2. The frame elements assume a plain uniaxial stress and strain state, and stirrups are neglected. Shear failure can not be detected.

2.2.4 Torque

Of all six internal forces torque exhibits the most sophisticated structural behavior. It is distinguished between equilibrium and compatibility torsion. The latter is often neglected in structural analyses. Due to cracking a beam elongates under pure torsion. The bearing mechanism can be idealized by a spatial truss model. Three main failure modes occur: collapse of the concrete strut, yielding of the stirrups, or yielding of the longitudinal reinforcement. For the selected beam VS2 [7] the reinforcement is governing.

In Fig. 4 torque is plotted versus twist. For a description of the three-dimensional finite element model the reader is referred to [8]. Due to the linearized stress-strain relation in A the torsional...
stiffness is overestimated at first and then decreases too abruptly. For large deformations the behavior can not be adequately predicted. Plot B perfectly fits in the first range, but diverges for large deformations. The frame elements of C are restricted to plane systems. Program D neglects the nonlinear effect for torsion and assumes an ideally linear behavior. The reduction of the torsional stiffness is coupled to the reduction of the bending stiffness, which is constant here.

![Fig. 5: Buckling](image1)

![Fig. 6: Frame](image2)

### 2.2.5 Buckling of columns

For the previous systems the material nonlinearity only has an influence on the deformations. For a column under eccentric axial load the geometrical nonlinearity and its influence on the internal forces have to be considered. As an example the authors choose column II.1 [9].

In Fig 5 the green curve A corresponds to the exact theoretical solution of the elastic second order theory up to the kink. The kink is caused by cracking. Compared to the experimental curve the behavior is too soft in the first range and too stiff in the second range, and the ultimate load is overestimated. B, C, and D show adequate results. The relatively good accuracy of all programs can be explained by the dominating geometrical influence, which proves to be captured well by all programs. The elements of C are geometrically exact even for large deformations.

### 2.2.6 Frame

For statically indeterminate structures the distribution of internal forces depends on the distribution of stiffness. Thus, the effective stiffness has a great influence on the system behavior. The redundancy allows for a redistribution of forces after a local failure of a member. The analysis is run for frame 2D18 [10]. The final failure occurs in the concrete at the upper ends of both columns. The load-displacement plot can be seen in Fig. 6.

All four analyses show a too steep ascent in the initial, uncracked range. However, in the following curve A draws near the experimental curve and approximates the ultimate load satisfactorily. Model B overestimates the stiffness but underestimates the ultimate load. Model C also misses the exact stiffness but fits the ultimate load in a better way. The computation of D ends before the typical failure behavior, including rapidly increasing displacement, occurs.

### 2.2.7 Beam grid

Another example for indeterminate structures are the beam grid experiments by [11]. The girder is T-shaped in plane. The load is applied to the longitudinal beam (Fig. 7). Bending moment, torque, and shear force act concomitantly in the cross beam. In the chosen S5 experiment failure is caused by crushing of the compression zone under the loading point.

All analyses show poor results. A and D take a purely linear course before the computations end. The linear curves may be explained by the linearized constitutive law of A and the constant torsional stiffness of D, respectively. B overestimates the stiffness but estimates the stiffness reduction well, and it captures the ultimate load to some extent. C is limited to plane systems.
2.3 Evaluation of results

Table 1 summarizes the evaluation of the benchmark series. None of the programs is able to analyze all experiments satisfyingly. There is no clear tendency to overestimation or underestimation of the stiffness and the bearing capacity. The results do not answer the question whether frame or three-dimensional elements are more adequate for the modeling of reinforced concrete frames. The assumptions which are made within beam theory provide a good approximation. In contrast, 3D models require a high effort in modeling and computation, and difficulties remain in the definition of boundary conditions and the reinforcement, so frame elements seem to be more appropriate. Using frame elements, results are available directly in form of internal forces, which are more convenient and better understood by practicing engineers than stress or strain values. However, so far no general accepted model exists which considers material nonlinearities for all six internal forces. Therefore in section 3 a new hybrid approach is presented.

Perfectly accurate curves may be achieved by an adjustment of the material parameters. In particular the choice of tension stiffening, shear retention or viscosity parameters has a significant influence. For three-dimensional constitutive laws additional parameters are required. The experimental determination of all these parameters is difficult. For these parameters sensitivity analyses would be reasonable. Often the experiments available in literature are not described in details. The documentation is often restricted to load and displacement measurements. A precise interpretation of the deviation between experiment and analysis is therefore not possible. On this account a series of experiments with RC frames is planned at the institute of the authors.

In a next step the benchmark could be extended to time depending effects. In [12] a benchmark is presented for creep and shrinkage. Another possibility is the extension to a multiaxial stress state. Thereby the well known experiments of [13] and [14] can be applied.

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
A & - & - & + & - & o & + & - \\
B & + & - & + & o & + & - & o \\
C & + & - & n/a & n/a & + & o & n/a \\
D & o & + & n/a & - & + & - & - \\
\end{array}
\]
3. Cross-sectional analysis

The distinct material nonlinearity of frame elements can be captured at cross-sectional level. A cross-sectional analysis determines the strain state corresponding to given sectional forces. It can be applied to the computation of interaction diagrams and force strain relationships such as moment-curvature curves. The main application, however, is the nonlinear structural analysis of spatial frames. Existing cross-sectional models can be classified into resultant models, truss models, uniaxial fiber models, wall models as well as models based on finite element analyses. Resultant models describe the cross-sectional behavior directly in terms of internal forces and generalized strains. They are completely empirical or the result of a detailed cross-sectional analysis. Truss models were the first models to describe the behavior of RC frames. Current design codes still base on these models but truss models apply only to the ultimate limit state. Uniaxial fiber models are the most common nonlinear cross-sectional approach for frame elements. They are generally restricted to biaxial bending and normal force. Wall models discretize the cross section into coupled membrane elements. A locally plane strain and stress state is assumed and thus transverse forces and torsion or, more precisely, St. Venant torsion, can be covered. However, so far there is no generally accepted model for arbitrary cross-sections under spatial section forces.

![Fig. 8: Cross-Sectional Analysis](image)

A new hybrid approach is proposed. The approach combines a uniaxial fiber model with a wall model. The cross section is divided into two components. The stirrups, the adjacent concrete and longitudinal rebars form a notional, thin-walled cross section assembled from membrane elements. The second component comprises the remaining concrete and longitudinal reinforcement; a uniaxial stress-strain state is assumed (Fig. 8). The uniaxially stressed areas only contribute to biaxial bending and normal force, the membrane elements additionally bear torque and transverse forces. The strain state of the cross section is described by generalized strain measures, their longitudinal derivatives, and additionally by the transversal and shear strains of each membrane element. It is determined by equations of equilibrium, continuity and the constitutive laws. The nonlinear problem is solved in four levels by a damped Newton-Raphson method. For a detailed description of the approach see [15]. The approach allows for a unified design procedure.

4. Unified design procedure

In the conventional design procedure initially the internal forces are determined by a linear elastic structural analysis. Afterwards the cross sections are designed for the resulting internal forces. In contrast to the computation at system level, the design of the cross section normally takes into account material nonlinearities. The contradiction between the linear determination of internal forces and the nonlinear design of cross sections is partially resolved by the introduction of the new generation of European reinforced concrete codes like [4] and [16]. Their safety concept is based on a semiprobabilistic approach. A nonlinear analysis at both system and cross-sectional level now constitutes a standard design procedure. According to these codes, the internal forces are computed in a nonlinear analysis based on the mean values of the material properties. Thereby the introduction of a global safety factor on the resistance side is an important step towards a consistent safety format for the nonlinear analysis. The new European codes propose a constant safety factor of approximately 1.3 independent of the cross-sectional properties [17]. The required safety level is maintained by a reduction of the theoretical mean value of the concrete strength [18]. The constant
global safety factor simplifies a nonlinear analysis substantially, because the ultimate load level does not depend on the type of cross-sectional failure, and the same material laws apply for the determination of the internal forces and the design of the cross section.

The interaction between cross section and system level is now considered in a more consistent way. However, the interaction is limited to biaxial bending and normal force. Inconsistencies also remain at cross-sectional level. The design for biaxial bending and normal force is carried out by limiting the strains under the assumption of a purely uniaxial stress state. In contrast, the limit states for transverse forces and torsion are defined in terms of stresses and are based on idealized truss models. These models are only suited for the ultimate limit state, since they presume the concrete to be cracked. The compatibility is controlled by the choice of the angle of the compressive strut. The combined effect of torsional and shear forces is considered only empirically. Interactions between bending and normal forces on the one hand and transverse and torsional forces on the other hand are only partially considered, e.g. the influence of the torsional moment on the bending stiffness is neglected.

The hybrid cross-section model presented in section 3 allows for a unification of the design procedure. For each membrane element in the cross section a biaxial strain state compatible to all six internal forces is determined [8]. The limit state for all force components is defined in terms of principal strains. Whether the strain limits of the current uniaxial design procedure also apply to the new biaxial model has to be checked. In the serviceability limit state the limitation of the stresses and the crack width also may be substituted by a limitation of the strains, either in form of smeared or local strains.

5. Conclusions

Despite progress in the development of analysis software, problems remain concerning their general application. A benchmark series consisting of seven exemplary frame structures displays several deviations from experimental data. The benchmark is run with both three-dimensional and frame elements. It reveals the necessity of an approach capturing the nonlinearities and interaction of all six internal forces. Consequently a new model for the cross-sectional analysis is suggested. The hybrid approach combines a uniaxial fiber model with a wall model consisting of membrane elements. It provides the opportunities of a unified design procedure. The procedure limits the principle concrete strains and the reinforcement strain produced by all six internal forces. Hence the determination of the internal forces and the design of reinforced concrete frame structures are reduced to a single nonlinear analysis.

6. References


