Continuous state feedback control for twin rotor damper

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ABSTRACT: The twin rotor damper is a recently developed active mass damper. Its feasibility has already been proven in principle, and its ability to damp structural vibrations, e.g. wind-induced or pedestrian-induced bridge vibrations, has been demonstrated [1]. Two eccentric masses rotate with the same angular velocity about parallel axes generating a dynamic control force that can be used for damping vibrations. Various control algorithms can be applied. For example, the masses can rotate with a constant angular velocity that is set to a natural circular frequency of the system. The phasing to the vibration of the structure is chosen such that the control force opposes the velocity of the system. The correct phasing is checked and corrected only once per half cycle (closed loop control with large time intervals). Alternatively, the target position of the rotors can be determined and set continuously (closed loop control with short time intervals). Such control algorithm can be preferable when the motion to be damped is non-harmonic. However, this leads to a rotor velocity that is not constant but variable. The degree of variability should be kept small to ensure an efficient mode of operation. A continuous state feedback control is developed for this purpose.

KEY WORDS: Structural vibrations; Active structural control; Active damping; Twin rotor damper.

1 INTRODUCTION
1.1 Twin rotor damper

The basic unit of the twin rotor damper (TRD) consists of two eccentric masses (control masses) that rotate about two parallel axes (Fig. 1) [1]. In one operational mode, both eccentric control masses rotate in opposite directions with a constant circular control frequency \(\omega_c\) (= rotor speed = angular velocity) that is set to the natural circular frequency of the system. If, as indicated in Fig. 1, the control position \(\phi(t)\) of both rotors is also the same, a vertical harmonic control force \(f_c(t)\) is generated that can be used for damping. The control force \(f_c(t)\) is given by the expression

\[
f_c(t) = m_r \omega_c^2 \cos(\phi(t))
\]

where \(\phi(t)\) = the time-dependent control position, \(m_r\) = the total control mass, and \(r_c\) = the control radius. The dot operator indicates differentiation with respect to time.

\[
\phi(t) = \omega_c t + \phi_0
\]

where \(\phi_0\) = the initial damping phase. In absence of external excitation the equation of motion of the system is given by

\[
(m + m_c) \ddot{x} + c \dot{x} + kx = m_r \omega_c^2 \cos(\omega_c t + \phi_0)
\]

where \(m + m_c\) = the total mass of the system, \(c\) = the damping constant, and \(k\) = the stiffness. The optimal initial damping phase \(\phi_0\) depends on the initial conditions, in particular, on the initial velocity and the initial displacement of the system. For the particular initial conditions,

\[
x(t = 0) = x_0 > 0 \quad \text{and} \quad \dot{x}(t = 0) = \dot{x}_0 = 0
\]

the optimal initial damping phase \(\phi_0\) is \(-\pi/2\) for systems without structural damping. Further information on the twin rotor damper, especially the determination of the optimal initial damping phase, can be found in [1]. The described mode of operation leads to a low power demand of the twin rotor damper shown in [1] or [2].

1.2 Objective

In this paper a continuous feedback control for the TRD is presented. To do this, we compute the position of the system, which is an observable state. Please note that the position of the system is not the displacement \(x(t)\), and this position will be defined in the following chapter. With the continuously measured position of the system, a target position \(\phi_{tar}(t)\) is computed in such a way that the resulting control force opposes the velocity of the system. A controller will ensure that the control position \(\phi(t)\) achieves the target position \(\phi_{tar}(t)\) in a short time. The difference between the control position and the position of the system is the decisive point for the continuous feedback control and the resulting oscillations of
the rotor speed. This position difference is studied in chapter 2.

2 DIFFERENCE BETWEEN CONTROL POSITION AND POSITION OF SYSTEM

2.1 Neglecting structural damping

Neglecting structural damping, but introducing the mass ratio \( \mu_c \) and the natural circular frequency of the system \( \omega_n \), which are

\[
\mu_c = \frac{m_c}{m_c + m} \quad \text{and} \quad \omega_n = \sqrt{\frac{k}{m_c + m}}.
\]

Eq. 3 can be rewritten

\[
\ddot{x} + \omega_n^2 x = \mu_c r_c \omega_c^2 \cos(\omega_c t + \phi_0). \tag{6}
\]

The displacement response \( x(t) \) is obtained by solving Eq. 6 analytically, for which the circular control velocity \( \omega_c \) is set to the natural circular frequency of the system \( \omega_n \)

\[
x(t) = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t + \ldots
\]

\[
-\frac{1}{2} \mu_c r_c (\omega_n t \sin(\omega_n t + \phi_0) - \sin \phi_0 \sin \omega_n t)
\]

Differentiating with respect to time yields the velocity response of the system \( \dot{x}(t) \) and further differentiating yields the acceleration response of the system \( \ddot{x}(t) \) [1].

![Fig. 2: Normalized displacement response of the system, initial conditions according to Eq. 4, \( c = 0, \mu_c r_c/x_0 = 0.05 \) and \( \phi_0 = -\pi/2 \).](image)

Fig. 2 shows the normalized displacement response \( x(t) \) of the system [1]. The time axis is normalized to the natural period

\[
T_n = \frac{2\pi}{\omega_n}.
\]

Up to the point in time \( t_{cs} \) the control force provides positive active damping to the system [1]. Positive and negative displacement peaks decay linearly. After the point in time \( t_{cs} \) the control force excites the system giving negative active damping to the system [1]. This paper focuses on the time range in which positive active damping is provided by the control force \( t < t_{cs} \). In addition, a control method is presented that enables a continuous state feedback control for this time duration. Two additional states of the system to describe the progress of the system in one vibration cycle are introduced and we term these additional observable states positions of the system

\[
\psi_1 = \arctan 2(\dot{x}(t)\omega_n, \ddot{x}(t)) \quad \text{and} \quad \psi_2 = \arctan 2(\ddot{x}(t)\omega_n, \ddot{x}(t)).
\]

The arctan2 is the arc tangent function, which uses two inputs as arguments to additionally obtain information about the quadrant. Positions of the system range in radians and have a value range from \(-\pi\) to \(\pi\). Differences between the control position \( \phi(t) \) (Eq. 2) and the positions of the system (Eq. 9) are

\[
\alpha_1 = \phi(t) - \psi_1(t) \quad \text{and} \quad \alpha_2 = \phi(t) - \psi_2(t),
\]

and we term \( \alpha_1 \) and \( \alpha_2 \) position differences.

The control position \( \phi(t) \), the positions of the system \( \psi_1(t) \) and \( \psi_2(t) \), and the positions differences \( \alpha_1(t) \) and \( \alpha_2(t) \) are in radians. The control position \( \phi(t) \) or the positions of the system \( \psi_1(t) \) and \( \psi_2(t) \) can be added or subtracted by arbitrary multiples of \( 2\pi \). This can also be done with the position differences \( \alpha_1(t) \) and \( \alpha_2(t) \) and this is done in such a way that they have a value range from 0 to \( 2\pi \) (Fig. 3).

![Fig. 3: Position differences \( \alpha_1(t) \) and \( \alpha_2(t) \), \( c = 0 \), initial conditions according to Eq. 4, \( \mu_c r_c/x_0 = 0.05 \) and \( \phi_0 = -\pi/2 \).](image)

Fig. 3 shows the position differences \( \alpha_1(t) \) and \( \alpha_2(t) \) for the described operational mode. Up to the point in time \( t_{cs} \) position difference \( \alpha_1(t) \) is nearly \( \pi/2 \) and it coincides the constant value \( \pi/2 \) at those points in time where positive and negative displacement peaks occur. In the time intervals between two displacement peaks the position difference \( \alpha_1(t) \) slightly falls below the value \( \pi/2 \) for \( t < t_{cs} \). After the point in time \( t_{cs} \) the position difference \( \alpha_1(t) \) slightly exceeds the constant value \( 3\pi/2 \). A similar behavior can be identified for the course of the position difference \( \alpha_2(t) \). For \( t < t_{cs} \) it coincides with the value \( \pi \) at the points in time displacement peaks occur exceeding it slightly between two successive peaks. After the point in time \( t_{cs} \), it is slightly below the value \( 3\pi/2 \) between two successive peaks. For both position differences \( \alpha_1(t) \) and \( \alpha_2(t) \), it can be stated that the deviations from their constant values increases with smaller amplitudes. This can be clearly seen in Fig. 4, in which only one vibration
cycle is plotted for different values $\mu_{cr}/x_0$. Displacement peaks are indicated by crosses. It can be seen that the deviations from a constant value approach zero if the value $\mu_{cr}/x_0$ goes to zero. The higher the value $\mu_{cr}/x_0$ is the more the position differences $\alpha_1(t)$ and $\alpha_2(t)$ deviate from the value $\pi$ or $3/2\pi$ in Fig. 4 the maximum deviation from the constant position difference $\pi/2$ is approximately 0.027, which corresponds to $1.5^\circ$. A very similar tendency can be identified for the position difference $\alpha_2(t)$ shown in Fig. 5 but the values for the position difference $\alpha_2(t)$ exceed the constant value $\pi$ between two successive displacement peaks $t<t_{cs}$.

The maximum deviations of the position differences $\alpha_1(t)$ and $\alpha_2(t)$ from their constant values indicated by the $D_1$ in Fig. 4 and by $D_2$ in Fig. 5 for $\mu_{cr}/x_0 = 0.05$ are approximately 0.027 and 0.028 radians. The absolute value of the deviations from $\pi$ or $\pi/2$ are approximately equal; however, the deviations have opposite signs. Comparing Fig. 4 with Fig. 5, a similar conclusion can be drawn for different $\mu_{cr}/x_0$. Therefore, we also compute the sum of the position differences

$$\alpha(t) = \alpha_1(t) + \alpha_2(t) = 2\phi(t) - \psi_1(t) - \psi_2(t).$$

The reason for computing this will be clarified in the following discussion (chapter 3). Fig. 6 shows the sum of the position differences $\alpha(t)$ for one vibration cycle. In one vibration cycle it exceeds its constant value $3/2\pi$ two times and also falls below its constant value $3/2\pi$ two times. Compared to the maximum deviation $D_2$, the absolute value of the deviation is reduced by 98% to a maximum deviation $D_{sum}$ (Fig. 6) of about $5.0\times10^{-4}$.

In this subsection we discovered that the position differences oscillate close to a specific constant value in the specified operational mode for $t<t_{cs}$. This simple fact is essential for the continuous feedback control presented below. Moreover, we discovered that by superposing these two position differences the oscillations are significantly smaller than the oscillations of the individual position differences $\alpha_1(t)$ or $\alpha_2(t)$.

2.2 Considering structural damping

In this subsection we consider structural damping ($\zeta > 0$). The damping ratio is defined by

$$\zeta = \frac{c}{2(m + m_c)\omega_n}.$$  

If structural damping is present, the circular control frequency $\omega_c$ is set to the damped natural circular frequency of the system $\omega_d$ [1], which is defined in standard literature, e.g., [3] and [5]. From Eq. 3 it follows the canonical form

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = \mu_{cr}\alpha_c^2\cos(\omega_c t + \varphi_0).$$

The optimal initial phase $\varphi_0$ for systems with structural damping and for the initial conditions according to Eq. 4 can be computed by the following equation [1]

$$\varphi_0 = \pi + \arctan\left(\frac{2\sqrt{1-\zeta^2}}{\zeta}\right) \text{ for } x_0 > 0.$$  

Following the same procedure as in the previous subsection leads to the position differences $\alpha_1(t)$ and $\alpha_2(t)$ when structural damping is present. The natural circular frequency $\omega_n$ in Eq. 9
is replaced by the damped natural circular frequency $\omega_d$. Fig. 7 and Fig. 8 show the position differences $\alpha_1(t)$ and $\alpha_2(t)$ for different damping ratios. The time axes are normalized to the damped natural period

$$T_d = \frac{2\pi}{\omega_d}. \quad (15)$$

Each time a displacement peak occurs the position difference $\alpha_1(t)$ coincides with the constant value $\alpha_{1c}(\zeta)$.

$$\alpha_{1c}(\zeta) = \arctan \left( 2 \sqrt{\frac{1-\zeta^2}{\zeta^2}} \right). \quad (16)$$

From Fig. 7 it can be concluded that the higher the damping ratio $\zeta$ is the more the constant values $\alpha_{1c}(\zeta)$ indicated by the dashed lines are reduced. Moreover, the deviation from the constant value $\alpha_{1c}(\zeta)$ is higher the larger the damping ratio $\zeta$ and the values $\mu_{rc}/x_0$ are.

$$\alpha_2(t) = \alpha_1(t) + \alpha_2(t). \quad (18)$$

It oscillates about the constant value

$$\alpha_c(\zeta) = 2 \arctan \left( \sqrt{\frac{1-\zeta^2}{\zeta^2}} \right) + \frac{\pi}{2}. \quad (19)$$

Fig. 9 shows the sum of the position differences $\alpha(t)$ for one vibration cycle when structural damping is considered. The curves coincide with the constant value (Eq. 19) each time a displacement peak occurs ($t = 0, t = 0.5 T_D$, and $t = T_D$ in Fig. 9). The deviations from the constant value increase with higher damping ratios and with higher values of $\mu_{rc}/x_0$ but the value range is much smaller than the value range of the individual position differences of $\alpha_1(t)$ and $\alpha_2(t)$.

In this subsection we determined that even though structural damping is considered, the position differences between the control position and the position of the system oscillate about a constant value in the specified operational mode. It can be pointed out that the deviation from its constant position of the
sum of the position differences \( \alpha(t) \) is significantly smaller than the deviation of the individual position differences \( \alpha_1(t) \) or \( \alpha_2(t) \).

3 CONTINUOUS FEEDBACK CONTROL FOR TWIN ROTOR DAMPER

We will now take the step towards the continuous feedback control for the twin rotor damper. We will only consider systems without structural damping \((c = 0, \text{Eq. 6})\). The same continuous feedback control can be applied for systems with structural damping as the position difference is also nearly constant. So far, we have assumed that the circular control frequency \( \omega_c = \dot{\phi} \) (= rotor speed) is constant. If the circular control frequency is not constant, the inertial forces of the control masses also act on the system. These inertial forces have to be taken into consideration when establishing the equation of motion. Eq. 6 then has an additional force term on its right hand side \([1]\) (Eq. 20). However, the influence of the inertial forces in the specified operational mode on the motion is small as the circular control frequency is nearly constant. The circular control frequency \( \omega_c(t) \) (= rotor speed) is the first derivative with respect to time of the control position \( \phi(t) \).

\[
\ddot{x} + \alpha_2^2 \dot{x} = \mu I \left( \dot{\phi}(t)^2 \cos(\phi(t)) + \dot{\phi}(t) \sin(\phi(t)) \right). \tag{20}
\]

Taking the position difference \( \alpha_2(t) \), it can be approximated in the specified operational mode for \( t < t_{cs} \) as follows (Fig. 3 for \( t < t_{cs} \) and Eq. 11)

\[
\alpha_2 = \phi(t) - \psi_2(t) \approx \pi. \tag{21}
\]

Eq. 21 says that the difference between the control position and the position of the system is approximately \( \pi \) in the described operational mode. The position of the system \( \psi_2(t) \) is measurable at all points in time. Renaming the control position \( \varphi(t) \) in Eq. 21 in target position \( \varphi_{tar} \) (Eq. 20) and solving Eq. 21 for \( \varphi_{tar} \) leads Eq. 22. It describes the target position depending on the position of the system \( \psi_2(t) \)

\[
\varphi_{tar}(t) = \alpha_2 + \psi_2(t) = \pi + \psi_2(t). \tag{22}
\]

Inserting Eq. 9 into Eq. 22 yields

\[
\varphi_{tar}(t) = \alpha_2 + \psi_2(t) \approx \pi + \arctan(2(x(t) \alpha_3, \dot{x}(t))). \tag{23}
\]

The displacement \( x(t) \) and the velocity \( \dot{x}(t) \) are measured continuously and are used to determine the target position \( \varphi_{tar}(t) \). This target position \( \varphi_{tar}(t) \) is compared to the actual control position of the TRD \( \varphi(t) \), which is also measured continuously (Fig. 11). The error \( e(t) \) is defined by the difference

\[
e(t) = \varphi_{tar}(t) - \varphi(t). \tag{24}
\]

This input for the computation of the control error \( e(t) \). We want the input of the controller \( ce(t) \) (Fig. 11) to be restricted to values between \(-\pi \) to \( \pi \) (Fig. 10) because the absolute deviation of the actual position from the target position cannot exceed \( \pi \) (half a turn). If the control error \( ce(t) \) is positive \((0 \ldots \pi)\), the target position \( \varphi_{tar}(t) \) lags behind the actual position and the TRD needs to decelerate.

Let us now turn to Fig. 10. If the system is set in motion by any initial conditions and \( f(t) \rightarrow 0 \), the target position would rotate counterclockwise with a rotor speed that is equal to the natural circular frequency of the system. In this case the position difference \( \alpha_2 \) is equal to \( \pi \) (Eq. 21) \([1]\). If the control force affects the motion, the target position \( \varphi_{tar}(t) \) would not rotate with a constant rotor speed due to the approximation we made in Eq. 22. The influence of the control force is the reason for the target position not to rotate with a constant rotor speed.

If the actual control position \( \varphi(t) \) is equal to the target position \( \varphi_{tar}(t) \), the control error would be zero. To minimize the control error the controller is tuned in such a way that the actual control position tracks the target position for a short time - indicated by a low transient response of the control error \( ce(t) \) (Eq. 24 and Eq. 25). The example target position \( \varphi_{tar}(t) \) in Fig. 10 indicates a point in time at which the target position precedes the actual position of the TRD \((0 < \varphi < \pi)\).

According to Eq. 22, the target position varies between 0 and \( 2\pi \) as the position of the system has a value range from \(-\pi \) to \( \pi \). The control position \( \varphi(t) \) has a value range between 0 and \( 2\pi \) (Fig. 1). Thus, the value of the error \( e(t) \) is in the range of 0 to \( 4\pi \). As mentioned in Section 2.1, we can add or subtract arbitrary multiples of \( 2\pi \) to the target position as well as to the position of the system \( \varphi_2(t) \). To ensure that the value of the control error \( ce(t) \) is in the desired range of \(-\pi \) and \( \pi \), appropriate multiples of \( 2\pi \) are added or subtracted from the error \( e(t) \). In this way, the rotors are driven to the closest target position by the controller (Fig. 10).

A similar method can be used to apply the position difference in Eq. 10 or the sum of the position differences (Eq. 12), which is not presented here. In general, it can be said that this method ensures that the controller drives the rotor to the closest target position.

For applying the specified operational mode the rotors have to be close by the target position and must nearly rotate with the natural circular frequency of the system. The controller...
continuously steers towards a target position \( \varphi_{\text{tar}}(t) \) whose derivative with respect to time is approximately the natural circular frequency of the system. Thus, both conditions for the specified operational mode are valid.

To minimize the control error \( c(t) \), a linear PD-controller is used [4]. The control error is a rotational position difference, which is the difference between the target position \( \varphi_{\text{tar}}(t) \) and the actual position \( \varphi(t) \) of the TRD and is measured in radians. The output of the controller \( \dot{\varphi}(t) \) is given by

\[
\dot{\varphi}(t) = K_P(c(t) + T_D c(t)). 
\]

(25)

where \( K_P \) is the proportional gain, and \( T_D \) is the derivative time. The controller is tuned by varying these gains. Increasing \( K_P \) leads to a smaller transient response of the error \( c(t) \), larger peak overshoot and oscillation [4]. The derivative time \( T_D \) can be used to change the damping ratio of the controller independently of the natural circular frequency of the controller [4]. In the following numerical simulations, it is assumed that the actuators achieve the angular acceleration \( \dot{\varphi}(t) \), which can be ensured by an internal motor controller.

Since the control error \( c(t) \) has the unit quantities and the output of the controller is the angular acceleration of the control position \( \dot{\varphi}(t) \), which has the unit rad/s², the proportional gain \( K_P \) has the unit 1/s², and the derivative time \( T_D \) has the unit seconds. Proportional- and derivative-gain are hand-tuned and constant (linear time invariant controller).

\[
\begin{align*}
\text{actual control position } \varphi(t) & \quad \text{Computation of target position } \varphi_{\text{tar}}(t) \\
\text{control error } c(t) & \quad \text{controller } \dot{\varphi}(t) \\
\text{control force } f_c(t) & \quad \text{TRD } x(t) \\
\end{align*}
\]

Fig. 11: Continuous state feedback control for twin rotor damper.

4 NUMERICAL SIMULATIONS

4.1 Assuming a constant difference between control position and position of system

Performing a simulation with the described feedback control (chapter 3) for different controller gains yields the response given in Fig. 12. The system is subjected to the initial conditions of Eq. 4. A target position is continuously computed and the rotors are driven to it by the controller. At the beginning of the simulation the control error \( c(t) \) becomes (relatively) large since the rotor speed is zero, and thus deviates from the desired quantity - target \( \omega_n(t) = \varphi_{\text{tar}}(t) = \omega_n \). First the rotor speed \( \omega_n(t) \) must increase to equal the natural circular frequency of the system \( \omega_n \) to achieve the specified operational mode where \( \omega_n(t) = \omega_n \). As can be seen from the Fig. 12, the rotor speed initially overshoots and then oscillates about the natural circular frequency of the system \( \omega_n \). After three vibration cycles the TRD has reached its specified operational mode signified by an almost constant rotor speed \( \omega_n(t) = \dot{\varphi}(t) \approx \omega_n \) and a small control error \( c(t) \) \((c < 0.1 \approx 5.72°)\). The resulting track of the actual control position \( \varphi(t) \) leads to a control force that has a direction opposite to that of the velocity of the system. It can be seen from Fig. 12 that the positive peaks as well as the negative ones of the displacement response decay almost linearly with time. The simulation is stopped before the vibrations come to rest since this paper is focusing on the time range \( 0 < t < T_n \).

![Image of Fig. 12: Simulation with initial conditions according to Eq. 4 \( \mu_{\alpha_F}/x_0 = 0.015, \omega_n = 2\pi 1/s, \) step size = 0.0001 s.

The simulation is also performed with increased controller gains leading to a controller which produces higher control effort. This means that the transient response of the control error is lower and the TRD achieves the specified operational mode quicker. However, when the TRD is operated in the specified mode \((4 \, s < t < 20 \, s)\), the occurring oscillations of the rotor speed are larger (Fig. 12).

Let us now turn to the unwanted oscillations of the rotor speed. As the position difference \( \alpha_2 \) is not exactly constant (Eq. 22), the resulting target position \( \varphi_{\text{tar}}(t) \) does not increase linearly, and thus the rotor speed \( \omega_n(t) \) is not constant. The oscillations of the rotor speed vanish if the amplitude of the vibrating structure approaches infinity or if \( \mu_{\alpha_F} \) approaches zero [1]. This is verified by this simulation since the oscillations of the rotor speed become larger with smaller amplitudes. Similar results are obtained if the position...
difference defined in Eq. 10 is used for control. In this case, the acceleration and the velocity are used as input to compute the target position. In a practical application, the acceleration signal is sensitive to disturbances and thus the target position \( \varphi_{tar}(t) \) may contain significant noise. Therefore, to avoid the high effort for filtering, it is recommended to use the approach described above.

4.2 Optimized computation of target position by considering the exact position difference

In section 4.1 it was determined that the assumption of a constant position difference leads to the unwanted oscillations of the rotor speed. Therefore, a target position \( \varphi_{tar}(t) \) is computed for which the exact position difference is considered. Eq. 10 gives the exact position differences.

Position difference \( \alpha(t) \) is

\[
\alpha(t) = \varphi(t) - \varphi_0 = \omega_n t - \arctan 2(x(t)\omega_n, \dot{x}(t))
\]

Solving Eq. 26 for \( \varphi(t) \), replacing \( \varphi(t) \) by \( \varphi_{tar}(t) \) yields

\[
\varphi_{tar}(t) = \varphi_0 + \omega_n t - \arctan 2(x(t)\omega_n, \dot{x}(t)) + \psi(t)
\]

where the first three terms describe the exact position difference with respect to \( t_{ref} \). This was approximated before (see Eq. 21). The fourth term \( \psi(t) \) is given by Eq. 9 and is measured continuously. The position differences are equal to the respective values \( \pi/2 \) and \( \pi \) at the points in time displacement peaks occur (Fig. 3). At those points in time we start to compute the exact position difference as a function of \( t_{ref} \). The following procedure may be used for a corresponding control algorithm:

- The initial value of the exact position difference is \( \pi \) (approximation as in the previous subsection).
- If a displacement peak is detected, the following parameters are set. The running time variable \( t_{ref} = 0 \), \( x_0 = x(t) \), \( \dot{x}_0 = 0 \), \( \varphi_0 = -\pi/2 \) for \( x(t) > 0 \) and \( \varphi_0 = \pi/2 \) for \( x(t) < 0 \), \( \mu, \nu, \) and \( \omega_n \) are known constants.
- Computation of the exact position difference with Eq. 26 as a function of the running time variable \( t_{ref} \).
- If \( t_{ref} > 0.5 T_n \), the position difference is approximated with \( \pi \). The reason for doing this will be explained in the following paragraph.
- A target position (Eq. 27) is computed continuously with the exact position difference and the continuously measured state \( \varphi(t) \).

Due to external excitation the actual period length can be shortened or elongated. Then, the next displacement peak occurs before or after the point in time \( t = 0.5 T_n \) indicated in Fig. 13 by black dots and due to the control algorithm the computed position difference jumps from its current value to \( \pi \) as the variable \( t_{ref} \) is set to zero when a displacement peak occurs. If the period length is shortened, meaning that a displacement peak occurs at points in time \( t_{ref} < 0.5 T_n \), the position difference jumps to the value \( \pi \) (see dotted line with arrows indicated by \( t_{ref} < 0.5 T_n \) in Fig. 13). These discontinuities can be avoided for \( t_{ref} > 0.5 T_n \) (displacement peak occurs at a later point in time) by approximating the position difference with \( \pi \) for \( t_{ref} > 0.5 T_n \). Therefore, in this case we compute the target position \( \varphi_{tar}(t) \) by

\[
\varphi_{tar}(t) = \varphi_0 + \omega_n t_{ref} - \arctan 2(x(t)\omega_n, \dot{x}(t)) + \psi(t)
\]

where \( \varphi(t) = rect\left(\frac{t_{ref} - T_n}{T_n}\right)\epsilon + \arctan 2(x(t)\omega_n, \dot{x}(t)) \) (28)

where \( \varphi(t) = rect\left(\frac{t_{ref} - T_n}{T_n}\right)\epsilon + \arctan 2(x(t)\omega_n, \dot{x}(t)) \) (28)

Simulations when applying the optimized computation of the target position (section 4.2 and section 4.3) are not plotted for brevity. The ramp up process \( t < 4 * T_n \) in Fig. 12) is quite similar and we are only taking into consideration the oscillations of the rotor speed \( \omega_n(t) \) about the natural circular frequency of the system \( \omega_n \) in the specified operational mode. Comparisons regarding this are made in section 4.4.
Displacement, velocity and acceleration are continuously measured, whereas $\omega_n$ is constant.

4.4 Comparison

In this section, the different possibilities to determine the target positions given above are compared regarding the oscillations of the rotor speed. For all simulations the same controller is applied. Corresponding time histories of the rotor speed $\omega_0(t)$ are plotted in Fig. 14. The time interval from 6 $T_n$ to 16 $T_n$ is of interest as in this time period the TRD is operated in the specified operational mode. To compare the results in terms of oscillations of the rotor speed $\omega_0(t)$, the standard deviation $\sigma$ is calculated. Results are listed in Tab. 1. Four cases labeled (1) to (4) are considered.

For case (2), for which a constant position difference between the control position and the position of the system is considered, the standard deviation of the rotor speed $\omega_0(t)/\omega_0$ is 2.4e-3. With the first optimization (case (3)), this value is reduced by 86% to a standard deviation of 3.2e-4. With the second optimization (case (4)), the standard deviation can even be reduced by 96.3% to a standard deviation of 8.8e-5. The latter two reductions are due to an improved computation of the target position (Fig. 11, section 4.2 and section 4.3).

On the other hand, oscillations of the rotor speed vary with the controller gains. Fig. 12 shows that the standard deviation of the normalized control speed is increased by 367% to a standard deviation of 1.1e-2 if both controller gains are doubled. In this case, however, the twin rotor damper approaches the desired rotor speed in a shorter time (lower transient response of the control error $ce(t)$).

The control of the TRD can be further improved - using controller gains $K_P$ and $T_D$ that depend on system states, e.g., the rotor speed $\omega_0(t)$ and the control error $ce(t)$. A useful modification may be the use of variable controller gains. For large errors, larger gains may be used to approach the desired control position and rotor speed in a short time. When the control error is smaller and the rotor speed nearly equals the natural circular frequency of the system, the gains may be reduced to lower the oscillations of the rotor speed.

5 CONCLUSIONS

In this paper a continuous feedback control for the twin rotor damper is presented. It is shown analytically as well as by simulations that the applied continuous feedback control leads to unwanted oscillations of the rotor speed, which are due to an approximation made in the closed-loop design. For example to minimize the required energy these oscillations should be kept small. This can be done by improving the computation of the target position. Two optimizations for the computation of the target position are presented. Numerical simulations prove that these procedures can reduce the oscillations effectively. In addition, it is shown that the controller yields favorable output when the controller gains are adapted.

REFERENCES