ACTIVE MASS DAMPERS FOR FLUTTER CONTROL OF BRIDGES

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ABSTRACT
In this study, the rotational mass damper and the movable eccentric mass damper for active bridge deck flutter control are investigated. A two-dimensional sectional model was used to describe the structural dynamics of a bridge deck subjected to wind. The flutter analysis of the uncontrolled and the controlled system was carried out using the flutter derivatives of a trapezoidal cross section, experimentally determined, and the Hurwitz criterion. A numerical simulation scheme in time domain was developed using suitable rational function approximations of unsteady aerodynamic forces. Several control scenarios were simulated. Furthermore, a coupled numerical-experimental simulation method was used for verification of the numerical results. The energy demand of the active control was considered as a main factor for the applicability of the proposed devices.

1. INTRODUCTION
Bridge deck flutter is a phenomenon of aeroelastic instability. The instable self-excited coupled, vertical and torsional vibration may cause collapse without advance warning. Until now, active flutter control for bridges has not been implemented in real structures. It can be assumed, however, that effective and robust active control devices are needed for future ultra-long-span bridges to reduce vibrations and to prevent flutter onset.

Active mass damper (AMD) devices change the dynamic properties of the bridge structure to enhance flutter stability. Miyata et al (1996) propose the application of robust control through an active twisting moment, which can be generated either by variable eccentric weight or by changing the rotational speed of a gyroscopic mass installed on the bridge deck. The variable eccentric mass damper has been further investigated by Wilde et al (1996).

In this study, two AMDs are investigated: the rotational mass damper (RMD) and the movable eccentric mass damper (MEMD). The equations of motion are established for a two-degree-of-freedom model of the bridge structure. An optimal linear static feedback controller is designed for flutter suppression. The motion-induced aerodynamic forces are considered in the analysis using experimentally obtained, frequency-dependent flutter derivatives. The flutter analysis of the uncontrolled and the controlled structure is carried out using the Hurwitz criterion. A time domain numerical simulation scheme has been developed using suitable rational function approximation (RFA) of the unsteady aerodynamic forces (Tiffany and Adams, 1987; Karpel, 1996). Various control scenarios under different flow regimes are simulated. A coupled numerical-experimental simulation method (Körlin et al, 2004) has been used to verify the numerical results. In bridge engineering the main barrier for implementation of the AMDs is the energy demand. Therefore, the energy consumption of the proposed devices is investigated.

2. AEROELASTIC STRUCTURE WITH ACTIVE CONTROL

2.1. Active mass dampers for bridge girders

2.1.1. Rotating mass damper

A two-degree-of-freedom aeroelastic model of the bridge structure is used. Fig. 1 shows the two degrees of freedom: the vertical displacement $h$ and
the rotational displacement \( \alpha \). An additional rotating mass is implemented in the center of the bridge girder for active control. The control variable, \( u \), is the rotational acceleration of damper mass. The equation of motion including the aero-dynamic forces, \( L_{ae} \), and \( M_{ae} \), can be written in matrix format:

\[
M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + K\mathbf{x} = F_{ae} + F_{c}u, \tag{1}
\]

where \( \mathbf{x}^T = (h/b, \alpha) \) is the displacement vector. The other vectors and matrices are defined as follows:

\[
M = \begin{pmatrix}
(m + m_c)b^2 & 0 \\
0 & I + I_c
\end{pmatrix},
\]

\[
C = \begin{pmatrix}
c_h b^2 & 0 \\
0 & c_\alpha
\end{pmatrix},
\]

\[
K = \begin{pmatrix}
k_h b^2 & 0 \\
0 & k_\alpha
\end{pmatrix},
\]

\[
F_C = \begin{pmatrix}
0 \\
L_{ae}b
\end{pmatrix},
\]

\[
F_{ae} = \begin{pmatrix}
L_{ae}b
M_{ae}
\end{pmatrix},
\]

where

\[
c_h = 2\zeta_h \omega_h m, \quad c_\alpha = 2\zeta_\alpha I \omega_\alpha, \\
k_h = m \omega_h^2, \quad k_\alpha = I \omega_\alpha^2,
\]

with the masses \( m \) and \( m_c \), the mass moments of inertia \( I \) and \( I_c \), the half of the bridge deck width \( b \), the structural damping ratios \( \zeta_h \) and \( \zeta_\alpha \), and the natural frequencies of motion \( \omega_h \) and \( \omega_\alpha \) of the vertical and rotational mode, respectively. The dot indicates differentiation with respect to time.

2.1.2. Movable eccentric mass damper

The active control is now exerted by a moment due to gravity \( g \) of the additional mass \( m_c \) (Fig. 2). The variable eccentricity \( u \) is the control variable. In this case, the equation of motion (Eq. 1) contains the following modified matrices:

\[
M = \begin{pmatrix}
(m + m_c)b^2 & m_cbu \\
m_cbu & I + m_cu^2
\end{pmatrix},
\]

\[
F_C = \begin{pmatrix}
0 \\
0
\end{pmatrix}.
\]

The mass matrix \( M \) is influenced by the control variable \( u \). The corresponding coefficients are time-varying. Thus, the equation of motion is non-linear. In this study, the stability verification by flutter analysis was carried out for a linearized system. The verification of stability of the nonlinear system does have general considerable difficulties. Therefore, it is a topic of further research. The system is linearized by neglecting the inertia effects due to damper mass \( m_c \) in the mass matrix. The bridge structure is controlled now by an external moment due to the gravitational force of the additional mass. Within the later described time domain simulation methods, the structural nonlinearities can be implemented as well. Thus, their effect on the control performance can be investigated.

2.2. Wind effects on bridge structure

The wind effects on the bridge deck consist of: motion-induced aerodynamic forces, the aeroelastic forces \( L \) and \( M \), and motion-independent aerodynamic forces due to gust or turbulence in the oncoming flow \( L_g \) and \( M_g \). In linear theory, the following superposition is valid:

\[
L_{ae} = L + L_g, \quad M_{ae} = M + M_g. \tag{5}
\]

Following the complex number approach of Starossek (1998), the motion-induced aerodynamic forces can be expressed in the following linearized form when constant-amplitude harmonic structural motion is assumed:

\[
\mathbf{F} = \begin{pmatrix}
Lb \\
M
\end{pmatrix} = \pi \rho v^2 b^2 \mathbf{Q} \mathbf{x}, \tag{6}
\]

with the matrix

\[
\mathbf{Q} := k^2 \begin{pmatrix}
c_{h}(k) & c_{\alpha}(k) \\
c_{ah}(k) & c_{\alpha\alpha}(k)
\end{pmatrix}, \tag{7}
\]

where \( \rho \) is the air density and \( v \) is the mean velocity of the oncoming flow. The non-dimensional reduced frequency \( k \) is defined as \( k = \omega b / v \), where \( \omega \) is the circular frequency of structural motion. The \( c_{mn} = c'_{mn} + j c''_{mn} \) are dimensionless complex flutter derivatives, where \( c'_mn \) and \( c''_{mn} \) are real numbers, and \( i \) is the imaginary unit.
This notation is based on the complex description of flutter derivatives by Klöppel and Thiele (1967), and it is preferred by the authors. The complex flutter derivatives are functions of the reduced frequency $k$ and depend on the cross section under consideration. They have been derived analytically by Theodorsen (1935) for a thin flat plate and can be determined experimentally by Theodorsen’s functions of the vertical and rotational modes, respectively.

The aerodynamic forces due to turbulence have been considered in the quasi-steady form (Simiu and Scanlan, 1996). In the present study, however, only the variation of the horizontal wind speed $v(t)$ is considered:

$$
\mathbf{F}_b = \left( \frac{L_g}{M_g} \right) = \pi \rho v b^2 \left( \frac{2C_L(\alpha)}{4C_M(\alpha)} \right) v(t). \tag{8}
$$

The $C_L(\alpha)$ and $C_M(\alpha)$ are the steady aerodynamic coefficients for vertical and rotational mode, respectively.

### 2.3. State space representation

#### 2.3.1. Frequency-dependent system matrix

The state equation of the aeroelastic system (see Eqs. 1, 6) reads

$$
\dot{z} = \mathbf{A}(k)z + \mathbf{B}_c u + \mathbf{B}_g v(t), \tag{9}
$$

where $z^T = (x^T, \dot{x}^T)$ is the state vector, and

$$
\mathbf{A}(k) = \begin{pmatrix}
0 & \mathbf{I} \\
-M^{-1}K_{ae} & -M^{-1}C_{ae}^k
\end{pmatrix},
\mathbf{B}_g = \begin{pmatrix}
0 \\
\pi \rho v b^2 M^{-1} \left( \frac{2C_L(\alpha)}{4C_M(\alpha)} \right)
\end{pmatrix},
\mathbf{B}_c = \begin{pmatrix}
0 \\
M^{-1} \mathbf{F}_c
\end{pmatrix},
$$

$$
\mathbf{C}_{ae}^{(k)} = \mathbf{C} - \pi \rho v b^2 k \begin{pmatrix}
c_{bh}(k) & c_{b\alpha}(k) \\
c_{ah}(k) & c_{a\alpha}(k)
\end{pmatrix},
\mathbf{K}_{ae}^{(k)} = \mathbf{K} - \pi \rho v b^2 k \begin{pmatrix}
c_{bh}(k) & c_{b\alpha}(k) \\
c_{ah}(k) & c_{a\alpha}(k)
\end{pmatrix}.
$$

The flutter derivatives, which were obtained from the experiments or from the Theodorsen’s function, are functions of the reduced frequency $k$ and are only determined for sinusoidal constant-amplitude motion. Therefore, the proposed state-space representation cannot be used for a purely numerical simulation of control scenarios with more general kinds of motions. In that case, a control model should be established in time domain using rational RFA of the unsteady aerodynamic forces.

#### 2.3.2. Time domain approximation

For the RFA of the unsteady aerodynamic forces the minimum state formulation of Karpel (1996) is used:

$$
\mathbf{Q}(k) \simeq \mathbf{A}_0 + ik \mathbf{A}_1 + (ik)^2 \mathbf{A}_2 + \mathbf{D}(ik \mathbf{I} + \mathbf{R})^{-1} \mathbf{E} (ik) \mathbf{k} . \tag{11}
$$

The coefficients of the matrices $\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2, \mathbf{D}$ and $\mathbf{E}$ are determined by a linear iterative least square optimization method. The lag terms in the diagonal matrix $\mathbf{R}$ are selected by a nonlinear non-gradient optimizer (Tiffany and Adams, 1987). The optimization procedure selected the elements of the matrices so that the RFA (Eq. 11) could fit the frequency dependent flutter derivatives (Eq. 7). The transformation into time domain is:

$$
\mathbf{Q} \dot{x} = \mathbf{A}_0 x + \frac{b}{\nu} \mathbf{A}_1 \dot{x} + \frac{b^2}{\nu^2} \mathbf{A}_2 \ddot{x} + \mathbf{D} \mathbf{x}_{ae}, \tag{12}
\mathbf{x}_{ae} = \frac{v}{b} \mathbf{R} \mathbf{x}_u + \mathbf{E} \dot{x}.
$$

The introduced vector $\mathbf{x}_{ae}$ contains the aerodynamic lag terms, which model the dynamic of the wind forces acting on the bridge deck.

Using the state vector $\mathbf{z}^T = (x^T, \dot{x}^T, \mathbf{x}_{ae}^T)$, the state equation is:

$$
\dot{z} = \mathbf{A} z + \mathbf{B}_c u + \mathbf{B}_g v(t), \tag{13}
$$

with the frequency-independent system matrix

$$
\mathbf{A} = \begin{pmatrix}
0 & \mathbf{I} & 0 \\
-M_{ae}^{-1} \mathbf{K}_{ae} & -M_{ae}^{-1} \mathbf{C}_{ae} & \pi \rho v b^2 M_{ae}^{-1} \mathbf{D} \\
0 & \mathbf{E} & -\frac{v}{b} \mathbf{R}
\end{pmatrix},
$$

where

$$
\mathbf{M}_{ae} = \mathbf{M} - \pi \rho v b^4 \mathbf{A}_2 ,
\mathbf{C}_{ae} = \mathbf{C} - \pi \rho v b^2 \mathbf{A}_1 ,
\mathbf{K}_{ae} = \mathbf{K} - \pi \rho v b^2 \mathbf{A}_0 . \tag{15}
$$

### 2.4. Control design

A linear static feedback control law $u = -\mathbf{k}^T z$ with the control parameters $\mathbf{k}^T = (k_1, k_2, k_3, k_4)$ is applied in Eq. 13. The state equation of the controlled system is given by:

$$
\dot{z} = (\mathbf{A} - \mathbf{B}_c \mathbf{k}^T) z + \mathbf{B}_g v(t). \tag{16}
$$

For control design, the disturbances due to $v(t)$ are neglected, the mean wind speed $v$ is assumed to be critical for the uncontrolled system. The
optimal control theory is used to dimension the control parameters $k_i$ \((i = 1, \ldots, 4)\). The cost function \(J\) is minimized to evaluate closed loop performance as follows:

\[
\min_k J, \quad \text{where} \quad J = \int_0^\infty (z^T Q z + u(t)^2) \, dt,
\]

with the weighting matrix \(Q\). The parameters \(k_j\) \((j = 1, \ldots, 4)\) can be determined by solving the matrix Riccati equation (Leipholz and Abdelk, 1986).

The control variable of the optimal control is assumed to be unlimited. Because of saturations in technical actuators, the control variable \(u\) is limited. However, for the flutter stability analysis of the controlled structure, small displacements are assumed. Thus, control saturations do not affect the result of flutter analysis. Their effect on control performance and on flutter stability under large structural displacements was studied using the proposed time domain methods (Sec. 3.2).

3. FLUTTER ANALYSIS AND TIME DOMAIN SIMULATION

3.1. Criterion of flutter stability

The flutter stability of the uncontrolled and the controlled aeroelastic system has been investigated analytically. This analysis is based on Eq. 9 omitting, however, the disturbance term due to \(v(t)\). The determination of critical wind speed can be conducted by a complex eigenvalue analysis (Starossek, 1992). Instead of considering the eigenvalue problem, the Hurwitz stability criterion is used in this study. It is based on the coefficients of the characteristic polynomial:

\[
det[\lambda I - (A^{(k)} - B k^T)] = \ldots
\]

\[
\ldots a_4 \lambda^4 + a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0.
\]

According to the Hurwitz criterion, the closed loop has an asymptotic stability if all coefficients \(a_i\) \((i = 0, \ldots, 4)\) are positive, and if the first four main diagonal sub-determinants of the Hurwitz-matrix are positive. For sake of brevity, the representation of the coefficients has been left out here. In Körlin et al (2004) these coefficients and a detailed description of the Hurwitz criterion are presented. The complex flutter derivatives (see Eq. 7) were determined experimentally using the forced vibration method (Thiesemann et al, 2003; Ukeguchi et al, 1966). In an iterative procedure, the wind speed was varied until the stability border was found. The procedure was repeated for modified reduced frequencies until the smallest, and governing, critical wind speed was identified.

3.2. Time domain methods

3.2.1. Time step integration method

The Eq. 16 was considered for numerical simulation of control scenarios in time domain. A 4th order Runge-Kutta method was implemented. A Davenport-spectrum (Davenport, 1972) with a characteristic length of 1200 meters and turbulence intensity (\(T_u\)) of 10% has been used to generate time series of the zero-mean stochastic turbulent component of the wind speed \(v(t)\).

3.2.2. Numerical-experimental simulation

A coupled numerical-experimental set-up has been used for simulation. The aerodynamic forces are measured on a scaled sectional model in a water channel. The structural and control forces are modelled numerically (Körlin et al, 2004). The sectional model is driven according to the displacements resulting from the numerical time-step integration of the equations of motion. The experimental investigation was carried out in the water channel of the Institute of Aerodynamics and Gas Dynamics (IAG), University of Stuttgart, Germany.

4. RESULTS

An erection stage of the longest suspension bridge in Europe, the Great Belt East Bridge, was considered. The structural parameters were taken from Walter (1994): \(f_h=0.099\)Hz, \(f_v=0.186\)Hz, \(b=15.5\)m, \(m=17.8 \times 10^6\)kg/m, and \(I=2.173 \times 10^6\)kg*m. The structural damping was set to zero. Only the rotational degree of freedom was taken into account to determine the control parameters (Eq. 17). The investigation was performed with a trapezoidal section model. The experimentally determined flutter derivatives can be found in Körlin et al (2004). The experimentally determined steady aerodynamic coefficients and their slopes are: \(C_L(\alpha)=0.01, C_M(\alpha)=0.08, dC_L(\alpha)/d\alpha=6.70\), and \(dC_M(\alpha)/d\alpha=1.27\). The RFA matrices (Eq. 11) with two aerodynamic lag terms are:

\[
A_0 = \begin{pmatrix}
-0.0537 & -1.7285 \\
0.0152 & 0.6752
\end{pmatrix},
\]

\[
A_1 = \begin{pmatrix}
-0.8989 & -0.7217 \\
0.6109 & -0.2984
\end{pmatrix},
\]
A_2 = \begin{pmatrix} -0.7213 & -0.0331 \\ -0.0786 & -0.0987 \end{pmatrix},
D = \begin{pmatrix} 0.3830 & -0.1759 \\ -0.0297 & 0.0173 \end{pmatrix},
E = \begin{pmatrix} -0.1530 & 4.0951 \\ 0.9642 & 5.4086 \end{pmatrix},
R = \begin{pmatrix} 0.2629 & 0 \\ 0 & 0.3583 \end{pmatrix}.

Assuming a mass ratio of \( m_c/m = 0.8\% \), the uncontrolled and controlled systems have been investigated for smooth (Tu\sim0\%) and turbulent (Tu=10\%) flow. Determined by flutter analysis, the critical wind speed are: 36 m/s for the uncontrolled structure (\( u=0 \)) and 50 m/s for the optimal controlled structure (\( u\to\text{opt} \)). In the corresponding numerical-experimental simulation, the onset of flutter vibration was observed at 38 m/s and at 49 m/s, respectively. Fig. 3 (a) shows the standard deviations of the structural responses plotted against the mean wind velocity. Considering the saturations in the control (\( u\to\lim \)), the critical wind speed has been reduced to be 43 m/s. The results of purely numerical simulations shown in Fig. 3 (b) are in good agreement to experimental results.

To implement an active flutter control in bridge engineering, the large energy demand is the main barrier. In Fig. 4, the energy consumption of the proposed AMDs during control scenarios with mean wind speed of 35 m/s and turbulence intensities (Tu) in the interval of 1\% to 20\% is plotted over the control level, which is defined by the turbulence intensity multiplied by the ratio of the increment of critical wind speed through the control and the uncontrolled critical wind speed. The mass ratio \( m_c/m \) has been varied from 0.5\% to 2\%. The dotted curves describe the increase of energy consumption with different damper masses and constant control parameters (Sec. 2.4). Because of the small ratio of the mass moments of inertia \( I_c/I \), the RMD performs large rotational accelerations and, consequently, has a high-level energy consumption. The MEMD requires energy to accelerate the damper mass horizontally. A lower energy consumption has been observed. Nevertheless, undesired horizontal motion would be induced by this device in a real three-dimensional bridge structure. Because of these disadvantages, the bridge deck flutter control problem can only be solved satisfactorily using the proposed devices in combination with linear control laws. Another AMD configurations and/or alternative control laws are needed to reduce the energy demand of active flutter control for bridges.

5. CONCLUSIONS

Linear optimal controlled mass dampers were investigated considering a two-dimensional sectional model of the bridge deck subjected to wind. The flutter analysis of the uncontrolled and controlled system was carried out using experimentally determined flutter derivatives of a trapezoidal cross section and the Hurwitz crite-
6. REFERENCES


