Modeling of Cone Penetration Testing with the Material Point Method

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1. Introduction

2. Material Point Method

3. Modeling Soil-Structure Interaction

4. Modeling Saturated Soil

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6. Summary and Outlook
Geotechnical problems encountered at Deltares require a numerical method which allows:

- consideration of large deformations of soil
- modeling of soil saturated / partially saturated with water
- use of sophisticated soil models to reproduce the complex behavior of different soils
- modeling of soil-structure interaction
Introduction

Examples of use of MPM for solving multi-phase problems

**Offshore**

Foundations: torpedo anchor (Al-Kafaji, 2013)

**Coasts**

**Rivers / Lakes**

**Groundwater**
Introduction

Examples of use of MPM for solving multi-phase problems

Offshore  Coasts  Rivers / Lakes  Groundwater

Foundations : spudcans (Beuth, 2013)
Introduction

Examples of use of MPM for solving multi-phase problems

Offshore  Coasts  Rivers / Lakes  Groundwater

Model test of the dumping of geocontainers (Deltares)

Water defenses: geocontainers (Hamad, 2013)
Introduction

Examples of use of MPM for solving multi-phase problems

Offshore  Coasts  Rivers / Lakes  Groundwater

Water defenses: dikes (Tan, 2013)
Introduction

Examples of use of MPM for solving multi-phase problems

Offshore

Coasts

Rivers / Lakes

Groundwater

Geohazards: soil liquefaction, scour, landslides

Soil liquefaction (Niigata earthquake, 1964)

Scour (Angus & Moore, 1982)

(Submarine) landslides (Goleta Slide, California)
Measurement of

- sleeve friction $f_s$
- tip resistance $q_c$
- pore water pressure $u$

Numerical analyses provide better understanding of mechanical processes and thereby more reliable estimates of soil properties.

Quasi-static problem as slow penetration rate of 2 cm / s
CPT involves large deformations of soil

Introduction

How to model:

- Large deformations?
- Soil-structure interaction?
- Saturated soil behaviour?

Centrifuge test (White, 2002)
2. Material Point Method
Material Point Method
Numerical analyses of large deformations

Updated Lagrangian FEM analyses give mesh distortions

Analysis of slope deformation

Analysis of pile installation

FEM: Remeshing when needed
Material Point Method

Numerical analyses of large deformations

in each step:

initial configuration  calculation step  after resetting the mesh

Lagrangian  Eulerian
Material Point Method

Numerical analyses of large deformations

Initial configuration

Deformed configuration
Quasi-static, implicit approach

\[ K\Delta u = F_{\text{external}} - F_{\text{internal}} \]

Dynamic, explicit approach

\[ M\ddot{u} = F_{\text{external}} - F_{\text{internal}} \]

- Incremental displacement vector
- Stiffness matrix
- Mass matrix
- Nodal acceleration vector
- External load vector
- Internal force vector

Material Point Method

Equilibrium equations
3. Modeling Soil-Structure Interaction

Considered approaches:

- Interface elements
- Bardenhagen contact algorithm
Though easy to use, conventional interface elements do not allow for large deformation analyses with FEM.
New interface stresses are computed after mesh resetting from consideration of equilibrium at interface nodes.
Modeling Soil-Structure Interaction

Sliding of elastic block

Elastic

Load

adhesion = 10 kPa

Applied Load [kPa]

20

1.0

Block displacement [m]
Modeling Soil-Structure Interaction

Sliding of elastic block

![Diagram showing the sliding of an elastic block with forces and displacements](image)

\[ \sigma_N \text{ [kPa]} \]

\[ +21.5 \]

\[ -21.6 \]

\[ \tau_s \text{ [kPa]} \]

\[ 10.0 \]
Modeling Soil-Structure Interaction

2. Bardenhagen contact algorithm

Predictor-corrector scheme

- predict velocities for $t + \Delta t$
- detect contact nodes for $t + \Delta t$

- approaching
  - sliding
  - normal component
  - to ensure no penetration
  - tangential component
  - to ensure sliding
  - no sliding
  - no correction

- separating
  - no correction
Modeling Soil-Structure Interaction

2. Bardenhagen contact algorithm: sliding block

Sliding for \( \tan(\alpha) = \mu \)  \( \mu = \) friction coefficient
4. Modeling Saturated Soil
Modeling Saturated Soil

1-phase

2-phase

Drained

Undrained

Partially drained

Saturated soil

Total stress analysis

Effective stress analysis
Modeling Saturated Soil

1-phase: undrained effective stress analysis

\[ \dot{\sigma} = \dot{\sigma}' + \dot{p}_{\text{excess}} \]

\[ \dot{\sigma}' = \mathbf{D}^{ep} \dot{\varepsilon} \]

\[ \dot{p}_{\text{excess}} = \frac{K_{\text{water}}}{n} \dot{\varepsilon}_{\text{vol}} \]

with \( \mathbf{D}^{ep} (E', \nu', c', \varphi', \psi') \)

taken from respective elastic-plastic soil model

Dissipation of excess pore pressures is not considered.
Modeling Saturated Soil

2-phase: v-w-formulation, continuous field equations

\[
\rho_f \dot{\mathbf{w}} = \nabla p - \mathbf{f}_d
\]

dynamic equilibrium of fluid

\[
(1-n) \rho_s \ddot{\mathbf{v}} + n \rho_f \dot{\mathbf{w}} = \nabla \cdot \mathbf{\sigma} + \rho_{sat} \mathbf{g}
\]
equilibrium of mixture (fluid + solid)

\[
\begin{align*}
n & : \text{soil porosity} \\
\dot{\mathbf{v}} & : \text{soil skeleton acceleration} \\
\dot{\mathbf{W}} & : \text{water acceleration} \\
\mathbf{f}_d & : \text{drag force} \\
\mathbf{\sigma} & : \text{total stress}
\end{align*}
\]
for nodes:

\[ M_w \ddot{w} + Q(w - v) = F_{w}^{ext} - F_{w}^{int} \quad \text{momentum of fluid} \]

\[ M_s \ddot{v} + \overline{M_w} \dot{w} = F_{w}^{ext} - F_{w}^{int} \quad \text{momentum of fluid and solid} \]

for material points:

\[ \frac{n}{K_w} \dot{p} = (1-n) \nabla \cdot v + n \nabla \cdot w \quad \text{mass balance} \]

\[ \dot{\sigma}' = D : \dot{\varepsilon} + \sigma' \cdot \dot{\omega} - \dot{\omega} \cdot \sigma' + (I : \dot{\varepsilon}) \sigma' \quad \text{stress-strain equation} \]

Vermeer, Jassim, Stolle, Van Esch (2010)
Modeling Saturated Soil

Validation v-w-formulation for Terzaghi 1D consolidation

$\sigma_y = -1 \text{ kPa}$

$T = \frac{c_v t}{h^2}$

Al-Kafaji (2013)
Modeling Saturated Soil

Application v-w-formulation to 1D consolidation (large strains)

Ceccato (2013)
5. CPT Analyses
CPT Analyses

First study with quasi-static MPM using implicit time integration

Interface elements placed along the penetrometer surface.
1-phase analyses with Tresca model.
CPT Analyses

Second study with dynamic MPM using explicit time integration

Bardenhagen contact algorithm extended for adhesion and adapted to 2-phase formulation.
1-phase- and 2-phase-analyses

Ceccato (2013)
CPT Analyses

Moving mesh approach

Moving mesh

Compressed mesh

Moving mesh

Compressed mesh
CPT Analyses

1-phase using Tresca: results of quasi-static MPM

Good agreement of results with:


Lu, Q, Randolph, MF, Hu, Y, Bugarski, IC. *A numerical study of cone penetration in clay*. Geotechnique, 2004
CPT Analyses

1-phase using Tresca: comparison dynamic and quasi-static MPM

\[ a = s_u \Rightarrow 13.4 \ (12.9 / 12.1) \]
\[ a = s_u / 2 \Rightarrow 11.8 \ (12.2 / 11.4) \]
\[ a = 0 \Rightarrow 10.2 \ (11 / 10.8) \]

Inertia effects have, as expected, no significant impact on results.
CPT Analyses

Comparison 1-phase and 2-phase undrained analyses

Excess pore pressures

1-phase formulation

2-phase formulation $k = 10^{-8}$ m/s
CPT Analyses

Comparison 1-phase and 2-phase drained analyses

Mean effective stresses

1-phase formulation

2-phase formulation

\[ k = 10^{-3} \text{ m/s} \]
Effective tip stress for drained penetration

Ceccato (2013)
1st study of cone penetration testing in undrained clay with
   - Quasi-static MPM using interface elements
   - 1-phase, Tresca model

2nd study of cone penetration testing in undrained clay with
   - Dynamic MPM using Bardenhagen contact formulation
   - 1- and 2-phase, Tresca model (ongoing work)

➢ Validation of 2-phase formulation for large deformations
➢ Extension of Bardenhagen contact formulation to adhesive contact and 2-phase formulation

Explicit MPM with:
   - multi-phase formulation (v-w)
   - contact formulation (Bardenhagen, interface elements)
   - sophisticated soil models (hypoplasticity, anisotropic model)

opens up new possibilities for numerical analyses of a broad range of quasi-static and dynamic geotechnical problems
Outlook

In collaboration with partners:

- Further development of MPM
  - Special visco-elastic-plastic plate elements (Tan, TU Delft)
  - Geomembranes (Hamad, University of Stuttgart)
  - Parallelisation (EU-FP7 MPM-Dredge)
  - Pre- and post-processing (EU-FP7 MPM-Dredge)

- Soil modelling
  - Softening models (Yerro, UPC Barcelona & Fern, University of Cambridge)
  - 3-phase formulation, unsaturated soil (Yerro, UPC Barcelona)
  - Erosion / sedimentation (Więckowski, TU Łódź, EU-FP7 GeoFluid & MPM-Dredge)

- Applications
  - CPT in partially drained conditions, e.g. silts (Ceccato, Padova University)
  - Pile jacking and driving in sand (Nguyen, TU Delft)
  - Wave impact on dikes (Tan, TU Delft)
  - ...

MPM-Dredge: University of Cambridge & dredging companies (Boskalis, Van Oord, DEME, Jan de Nul)
Outlook

Transition between soil and water

Soil with pore-fluid: Effective stresses
Water with soil-particles: Navier-Stokes
Outlook

Transition between soil and water

"Primeur" by Więckowski (2013)

non-dilatant soil
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