Source-Seeking and Level Curve Tracking in Complex Environment

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Abstract—In this paper we consider the problem of source seeking and level curve tracking in the presence of unknown obstacles or significant noisy measurements. This work is based on a previously proposed mission control strategy where a group of agents, based only on local measurements and neighbors’ information, is able to track and monitor an unknown scalar field. Here we present a novel solution for tracking a level curve while avoiding unknown obstacles in the agents’ path. By using a combination of a distributed localization method and time-varying formation control, a group of agents is able to overcome noise corrupted measurements and find the location of unknown source (field maximum). Simulated scenarios with a group of 32 agents illustrate the robustness of the proposed strategy for a time-varying field in the presence of obstacles and noise.

I. INTRODUCTION

The advantages of using Multi-Agent Systems (MAS) for complicated missions such as navigation and exploration of large and complex environments are broad. From fast coverage ability while avoiding human intervention, autonomous systems are becoming an efficient solution for complex tasks.

One topic in particular is monitoring and exploring disaster areas such as oil spills and nuclear meltdowns. Here we represent such problems as a generic mission control problem, where $\psi : \mathbb{R}^+ \times \mathbb{R}^p \to \mathbb{R}^+$ represents the (possibly time-varying) distribution of a relevant quantity (concentration of radioactivity, pollution, etc.) and $p$ is the dimension of the space to be explored. For exploring such a field, first the agents are required to locate the field’s maximum; this also known as the source-seeking problem and various solutions have been proposed. From using a single agent (see, e.g., [1]–[3]) for simplicity, to using a group of agents (see e.g., [4]–[6]) for enhanced exploration ability, the focus is on achieving better gradient estimation. An additional aspect of the field exploration mission is to track a specified level curve for exploring the field’s shape (see, e.g., [7]–[9]). The latter provides not only information on the location of the source but also on changes of the field’s structure. This paper is based on previous work reported in [10] where we propose a mission control strategy for exploring time-varying fields. The strategy contains a 4-tasks algorithm for locating the field’s maximum and tracking a specified level curve. The agents have to reach a discrete consensus deciding on the next task. The previous work focused on mission control under ideal conditions. In the current work, we extend this strategy to environments with obstacles and measurement noise. For instance, with obstacles in the region, the agents are required to continue their tasks while avoiding collisions. The topic of obstacle avoidance for multi-agent systems has been investigated thoroughly. For example, [11] offers distributed formation control and collision avoidance control laws for reaching consensus of a group of non-holonomic agents with limited communication. Each agent’s controller requires information only from its local neighbors. [12] proposes an optimal control approach that involves a non-quadratic penalty function for achieving consensus with obstacle avoidance capability. For providing a solution to the source seeking problem in a complex environment, [13] proposes strategies for adapting to real life environments such as obstacles and collision objects using Particle Swarm Optimization. The objective of the seekers is to communicate, and move in a manner so as to reach the global minimum of the cost function. One contribution of the current work is to provide the ability to avoid collision during the whole field exploration process. Thus, the agents avoid collision not only when seeking the source but also while tracking a level curve. A second contribution is providing noise robustness under corrupted measurements. [14] investigates robustness of consensus in networks driven by white noise and its dependencies on the graph topology, while [15] uses time-varying gains in the consensus protocol dependent on the measurement noise levels. In this paper we propose two possible methods: a noise robustness technique where the agents change their relative distance in relation to the noise level and a distributed localization method where based on shared information with other neighbors, an agent handles noise corrupted measurements by using a prediction technique based on a Kalman filter. Different ideas for using a distributed Kalman filter have been proposed (e.g., [16], [17]); here we use the idea of a central Kalman filter taking the average Kalman gain instead of the inverse-covariance and measurements as suggested in [18].

The rest of the paper is as follows. In Section II, the problem statement is introduced. Section III describes the 4-tasks strategy. In Section IV an obstacle-avoiding distributed control scheme is presented and in Section V a noise robustness techniques are described. Section VI presents simulation results with a group of agents in different scenarios. Finally, in Section VII conclusions are drawn.

II. PROBLEM STATEMENT

Consider a scenario involving the spatial distribution of a scalar quantity (e.g., concentration of a hazardous substance), which can be described by a scalar field. Let $G = (V, E)$ be an undirected and connected graph with $N$ agents and adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ defined as $a_{i,j} =$
The Laplacian matrix $L$ is defined as $L = \Delta - A$, where $\Delta = \text{diag}(A) - 1$ is the degree matrix of $G$ with diagonal elements of $d_i = \sum_j a_{ij}$ and $1 = [1, \ldots, 1]^T \in \mathbb{R}^N$. Let $r_i \in \mathbb{R}^2$ be the position of a single agent and let $\psi(t, r_i) : \mathbb{R}^+ \times \mathbb{R}^2 \to \mathbb{R}^+$ describe the time-varying concentration level of the field at $r_i$. Then, a group of agents that can measure the field and exchange information through a graph $G$ is supposed to find the field’s maximum, where we make the following assumption.

**Assumption 1:** $\psi$ has a global maximum in $r_m(t)$ and the velocity of the field, when time-varying, is constant.

In this paper, we consider a single-integrator model ($p = 2$) for the agents

$$\dot{r}_i(t) = u_i(t),$$
where $i = [1, \ldots, N]$, $r_i(t) = [r_i^x, r_i^y]^T$ is the position vector and $u_i(t) \in \mathbb{R}^2$ is the distributed control input.

Given a time-varying field $\psi(t, r_i)$ and a group of agents with arbitrary initial positions, the agents are required to explore the field (locate source, track level curve) while avoiding collision with unknown obstacles in the region. This shell achieved under noise corrupted measurements (in both field concentration and location) in different frequency ranges.

### III. Mission Control Strategy

This section describes briefly the 4-tasks algorithm proposed in [10] which the current work is based on. First, the agents search for the field’s maximum, then start searching for a specified level curve and finally divided into groups of anchors and patrols for monitor a time-varying field. Consider a two-level hierarchy graph, where a subgroup $G_1 = \{i \in [1, N_1]\}$ has a leader connected with other leaders in a higher hierarchy $G_2$, where in total $G = G_1 \times G_2$ (see [19]). First the leaders must agree on a selected task, using a discrete consensus protocol. At iteration step $l$, the switching vector is initialized with $S_t^{(0)} = [S_t^{(0)}, \ldots, S_t^{(0)}]$, $S_t^{(0)} \in \{1, 2, 3, 4\}$ and the iteration process is defined as

$$S_t^{(q+1)} = L \cdot S_t^{(q)},$$
where $L$ is the Laplacian matrix, $q \in \{0, \ldots, \bar{q}\}$, $\bar{q} = \max_{u,v} d(u,v)$ and $d(u,v)$ is the graph’s diameter. The selected task then is

$$P_l = [P_{1,l}, \ldots, P_{N,l}]^T,$$
$$P_{1,l} = \begin{cases} S_t^{(q)} & , S_t^{(q)} = 0 \\ P_{l-1} & , \text{else} \end{cases}, \quad P_0 = 1,$$
where $P_{l-1}$ is the previous result of the iteration process. For the switching conditions please refer to [10]. The distributed gradient estimation process is similar to [20], where approximation to Taylor series (neglecting high-order terms) leads to

$$\psi(t, r_j) = \psi(t, r_i) + (r_j - r_i)^T \nabla \psi(t, r_i),$$
where $j \in N_i$ and $N_i$ is the set of neighbors of agent $i$. The shared information is in the form of the computed slopes between agent $i$ and its neighbors in $N_i := \{i_1, \ldots, i_{|N_i|}\}$ (e.g. $i_1$ is the first neighbor $j$ of agent $i$).

$$b_i = [b_{i1}, \ldots, b_{iN_i}]^T \in \mathbb{R}^{|N_i|}, \quad b_i = \psi(r_{i_1}) - \psi(r_i)$$
$$R_i = [R_{i1}, \ldots, R_{iN_i}]^T \in \mathbb{R}^{|N_i| \times 3}, \quad R_i = [r_{i_1} - r_i]^T$$
and the gradient is computed according to

$$\hat{g}_i = (R_i^T R_i)^{-1} R_i^T b_i,$$
where $\hat{g} \in \mathbb{R}^{2N}$ and the inverse of $(R_i^T R_i)^{-1}$ exists iff $R_i$ is full column rank, which requires $|N_i| \geq 2$.

### IV. Obstacle Avoidance

In this section we propose a novel technique for avoiding collision with unknown obstacles by generating a virtual repulsive force around the obstacle using estimation of its center and influence radius.

**Assumption 2:** All obstacles are convex, bounded and time-invariant. The distributed control input for system (1) is

$$u_i = K_u \cdot e_i + \gamma_i,$$
where $K_u > 0 \in \mathbb{R}^{2 \times 2}$ is the same proportional controller for all agents. In addition, each agent is equipped with a distributed obstacle avoidance term $\gamma_i$, which is potential function based (Figure 1). Using a potential function as a repulsive force is a common and efficient technique but requires knowledge on the obstacle location and its influence radius (the radius of a circle around the obstacle), which in most cases this information is unavailable. Here, we propose a method for estimating an obstacle’s location and shape by generating a repulsive virtual potential field around the obstacle using a mean-run method. Let $o$ be an obstacle index where $o \in [1, N_{obs}]$ and $N_{obs}$ is the number of detected obstacles, and let $\rho_o$ be the agents’ sensing radius. At each $t_k = k \cdot T_s$, where $k, T_s$ are sampling instant and interval, respectively, an agent measures a location on the obstacle’s surface $r_{io}(t_k) = [x_{io}, y_{io}]^T_{t_k}$. The process is repeated in different locations where the obstacle’s shape is revealed. Once an obstacle is detected, $r_o(t_0) = r_{io}(t_0)$, then the process continues with estimating a virtual center

$$r_o(t_k) = (r_{io}(t_k) + r_o(t_{k-1}))/2,$$
and influence radius

$$h_o(t_k) = \max_{p \in [1, k]} ||r_o(t_k) - r_{io}(t_p)||$$
$$\rho_o(t_k) = \varepsilon_d \cdot \rho_o + h_o,$$
where $\varepsilon_d > 1$ is a tuning parameter. There are many applicable potential functions, here we choose one similar to [21],

$$f_o(\rho_o) = \left\{ \begin{array}{ll} \varepsilon_{obs}^2 (\rho_o^{-1} - \rho_o^{-1})^2 & , \rho_o \leq \rho_o \\ 0, & \text{else} \end{array} \right.$$
where $\varepsilon_{obs} > 0$ is a scaling factor and $\rho_{io} = ||r_i - r_o||$ is the relative distance between the $i$-th agent and the $o$-th
obstacle. Then, the repulsive force of obstacle \( o \) is given by the negative gradient of the potential function
\[
\gamma_{i,o} = - \nabla f_{o}(\rho_{i,o}) =
\begin{cases}
\varepsilon_{obs}(\rho_{i,o}^{-1} - \rho_{o}^{-1})\rho_{i,o}^{-3}(r_{i} - r_{o}) & , \rho_{i,o} \leq \rho_{o} \\
0 & , \text{else}
\end{cases}
\tag{11}
\]
Finally, the contribution of all obstacles are then calculated as
\[
\gamma_{i} = \sum_{o=1}^{N_{obs}} \gamma_{i,o}.
\tag{12}
\]

Using a potential function for obstacle avoidance might generate singularities. Let \( u_{ip} = K_{u} \cdot e_{i} \) and \( \gamma_{i} \) be the attraction and repulsive forces, respectively, and let \( \theta_{i} \) be the angle between them, i.e.
\[
\alpha_{i} = \frac{u_{ip}^T \gamma_{i}}{||u_{ip}|| \cdot ||\gamma_{i}||}, \quad \theta_{i} = \cos^{-1}(\alpha_{i}) \tag{13}
\]
then, a near-singularity arises when \( \theta_{i} \) is close to \( \pi \) (in other words, the forces sum up to a value small enough to cause an agent to stop moving). In this case we modify the control input as,
\[
u_{i}(t) = \begin{cases}
\nu_{i} + k_{s} \cdot \theta_{i} > \pi - \varepsilon_{o}, \quad \varepsilon_{i} = 1 \\
\nu_{i} \quad , \text{else}
\end{cases}
\tag{14}
\]
where \( k_{s} > 0 \in \mathbb{R}^{2}, \varepsilon_{o} > 0 \) are tuning parameters. This intervention is only active when an agent is inside the influence region of an obstacle, i.e.
\[
\varepsilon_{i}^{o} = \begin{cases}
1 \quad , \rho_{i,o} \leq \rho_{o} \\
0 \quad , \text{else}
\end{cases}
\tag{15}
\]

The error input to the controller is a composition of three parts - formation, gradient and obstacles, described by
\[
\varepsilon_{i}(t) = K_{f} \sum_{j=1}^{N} L_{ij}[(r_{f_{j}}(t) - r_{i}(t))] + K_{g}\gamma_{i}(r_{i}(t)) + K_{o,i},
\tag{16}
\]
where \( r_{f_{i}} = [r_{f_{j}}^{T}, r_{f_{j}}^{W}]^{T} \), \( r_{f_{j}} \) is the formation reference for agents \( i,j \) respectively, and \( L_{ij} \) is the \( i,j \) element in the Laplacian matrix. \( K_{f}, K_{g} > 0 \) are controller parameters for the formation and estimated gradient, respectively.

For the source seeking task, agents encircle obstacles using the gradient climbing element which continues pushing them to the maximum. This is not the case in the level-tracking task where a different solution is required. One solution is to set an arbitrary direction or excitation force for the agents to encircle the obstacle, but it requires a constant update regarding the desired direction, which becomes inefficient. Furthermore, it does not guarantee continuity of the specified level curve tracking task. Here we present a solution using the proposed field exploration strategy. As described in [10], in Task IV the agents are divided into two groups of patrols and anchors. Once an obstacle is detected by a patrol agent, the destination anchor becomes a temporary target which generates an attractive force for encircling the obstacle
\[
K_{o,i} = k_{o} \cdot d_{e}^{o} \cdot \varepsilon_{i}^{o},
\tag{17}
\]
where \( k_{o} > 0 \in \mathbb{R}^{2x2} \) and \( d_{e}^{o} \) is the distance between the patrol agent and the anchor target (according to the tracking direction)
\[
d_{e}^{o} = ||r_{a} - r_{i}||,
\tag{18}
\]
where \( r_{a} \) is the position of the target agent. The complete distributed control scheme is shown in Figure 2.

![Fig. 1: Agent \( r_{i} \) equipped with \( \rho_{d} \) sensing radios, detects obstacle \( o \)'s surface in several points \( r_{o} \in \mathbb{R}^{k+4} \) and estimates its virtual location \( r_{o} \) and the influence radius \( \rho_{o} \).](image)

![Fig. 2: Complete distributed control scheme for exploring a time-varying field in a complex environment](image)

Proposition 1: Consider the multi-agent system (1) with control law (7). Suppose that Assumptions 1, 2 and 3 are fulfilled. Then, for all \( r_{i}(0) \in \mathbb{R}^{p} \) and \( t \geq 0 \), the agents locate the unknown field’s maximum and a specified level curve with a bounded error of \( ||e|| \leq \varepsilon \), where \( e = [e_{1}, \ldots, e_{N}]^{T} \).

Proof: (Outline) The proof based on [10]. From equations (1) and (7),
\[
\dot{r} = \begin{bmatrix}
\dot{r}_{1} \\
\vdots \\
\dot{r}_{N}
\end{bmatrix} = K_{u} \cdot (-K_{f} L_{p}(r - r_{f}) + K_{g} \gamma(r) + K_{o} + \gamma),
\tag{19}
\]
where \( \gamma = [\gamma_{1}, \ldots, \gamma_{N}]^{T} \) is a vector of the obstacle avoidance elements. Due to the obstacle avoidance temporal characteristics, and under the assumption that the obstacles are convex and bounded (which enables the agents to encircle the obstacle), both source-seeking and level curve tracking share the same principle: that an obstacle’s repulsive force can be considered as a disturbance to the system that does not affect overall stability. Assuming that \( K_{u} \) has been absorbed in \( K_{f} \) and \( K_{g} \), we have
\[
\dot{r} = -K_{f} L_{p}(r - r_{f}) + K_{g} \gamma(r).
\tag{20}
The state equation is a superposition of the formation control and the gradient driven parts. The former is stable and guarantees consensus according to

$$\lim_{t \to \infty} r(t) = \frac{1}{N} H^T r(0) + r_f = r_c + r_f,$$  \hspace{1cm} (21)

where $r_c = \frac{1}{N}(r_1 + \cdots + r_N)$ is the average consensus position and $r_f$ is the formation offset. The latter drives the agents toward the maximum or to a reference level curve location, i.e., $\hat{g} = 0$. The combination of the two yields a geometric shape of the agents that includes the source, where the equilibrium point is inside the convex hull of the set of agents $r = [r_1, \ldots, r_N]^T$.

V. NOISE ROBUSTNESS

This section describes techniques to overcome noise corrupted measurements in both location and concentration level. Here we propose a distributed approach that can be used in field exploration missions.

A. Distributed Kalman Filter

First, we describe a solution for location estimation based on a distributed Kalman filter. An agent’s model for the Kalman filter is

$$r_{i,t_k} = Ar_{i,t_{k-1}} + Bu_{i,t_{k-1}} + w_{i,t_{k-1}},$$
$$z_{i,t_k} = Hr_{i,t_k} + v_{i,t_k},$$ \hspace{1cm} (22)

where $r_i \in \mathbb{R}^p$ is agent’s state vector, $z_i \in \mathbb{R}^m$ is the measurements vector and the variables $w,v$ represents the process and measurements noise respectively. They are assumed to be independent, white, and with normal probability distributions $p(w) \sim N(0,Q), p(v) \sim N(0,R)$ where $Q$ is the process noise and $R$ is the measurement noise covariance matrix. The process involves two steps; first, a prediction step using the agent model

$$\hat{r}_{i,t_k} = Ar_{i,t_{k-1}} + Bu_{i,t_{k-1}},$$
$$P_{i,t_k}^{-} = AP_{i,t_{k-1}}A^T + Q,$$ \hspace{1cm} (23)

where $P_{i,t_k}$ is the estimated error covariance matrix. The Kalman gain $K_i \in \mathbb{R}^{p \times m}$ minimizes the error covariance. Thus, averaging over agents’ neighbors generates a distributed Kalman gain which increases the model prediction accuracy according to

$$K_{i,t_k}^{-} = P_{i,t_k}^{-} H^T (HP_{i,t_k}^{-} H^T + R)^{-1},$$
$$K_{i,t_k} = \frac{1}{1 + |N_i|} \sum_{q=i,j \in N_i} K_{q,t_k}^{-},$$ \hspace{1cm} (24)

which can be equivalently expressed as

$$K_{i,t_k} = (A + \hat{I})K_{i,t_k}^{-} ||\Delta + \hat{I}||^{-1},$$ \hspace{1cm} (25)

with $K \in \mathbb{R}^{Np \times Nm}, A = \Delta \otimes I_{p \times m}, \Delta = \Delta \otimes I_{p \times m}, \hat{I} = I_{Np \times Nm}$. Finally, the correction step is

$$\hat{r}_{i,t_k} = \hat{r}_{i,t_k} + K_{i,t_k}(z_{i,t_k} - H\hat{r}_{i,t_k}),$$
$$P_{i,t_k} = (I - K_{i,t_k} H)P_{i,t_k}^{-}.$$ \hspace{1cm} (26)

B. Signal-to-Noise Ratio

Next, we describe a solution for noisy measurements of level curves, based on modifying the formation structure with respect to the signal-to-noise ratio. This way the gradient is measured in locations with a distance that is sufficient to result in a significant difference between measurements, which reduces the noise effect. Previous work conducted by [22] and improved by [23] presents a distributed consensus filter which improves the accuracy of time-varying signal tracking and attenuates high frequency noise. Assume that each agent measures a field signal corrupted by a zero-mean Gaussian noise $n_{\psi,i} \in \mathbb{R}$,

$$s_{\psi,i}(r,t) = \psi_i(r,t) + n_{\psi,i}(t).$$ \hspace{1cm} (27)

Then, the distributed low-pass filter is

$$\hat{\psi}_i(t) = \beta_{\psi} \left\{ \sum_{j \in N_i} a_{ij}(s_{\psi,i} - s_{\psi,j}) - \sum_{j \in N_i} a_{ij}(\hat{\psi}_i - \hat{\psi}_j) + (1 + d_i)(s_{\psi,i} - \hat{\psi}_i) \right\},$$ \hspace{1cm} (28)

where $\hat{\psi}_i \in \mathbb{R}^N$ is the estimated value of $\psi_i$ and $\beta_{\psi}$ is a scalar tuning parameter. Lower frequency noise (can be considered as a sequence of disturbances) is more difficult to filter. Therefore, measurements with significant differences are required. In the source-seeking task, by calculating the gradient in several relatively far locations, the agents are able to cope with noisy measurements and locate the maximum. From (6), the element of the estimated gradient between two neighbor agents is

$$b_{i,j} = s_{\psi,j} - s_{\psi,i} = (\psi_j - \psi_i) + (n_{\psi,j} - n_{\psi,i}) = \Delta \psi_{i,j} - \Delta n_{\psi_{i,j}},$$
$$R_{i,j} = r_j - r_i = d_{i,j},$$
$$\hat{g}_{i,j} = \frac{\Delta \psi_{i,j} - \Delta n_{\psi_{i,j}}}{d_{i,j}}.$$ \hspace{1cm} (29)

One can observe that once the distance increases, $\Delta \psi_{i,j}$ and $d_{i,j}$ are getting larger while $\Delta n_{\psi_{i,j}}$ remains the same. Thus, the varying range for $\hat{g}_{i,j}$ is getting tighter (i.e., more accurate). For this adaptive formation structure, we propose a technique which is based on a signal-to-noise ratio calculation. The agents change their group’s structure according to the level of concentration and noise (Figure 3). This way they will extend the search area when the SNR is low and narrow the formation when reaching the maximum.

![Fig. 3: Illustration of agents’ adapted formation](image-url)
The adaptive element is calculated by
\[ \Delta r_f = 20 \log_{10} \frac{S_{\psi, i}(t)}{\sigma_{\psi}}, \]
where \( \sigma_{\psi} > 1 \) is a tuning parameter for evaluating the noise level. The formation’s behavior depends on the ratio between the signal and the noise evaluation. If the ratio is less than 1 (i.e., \( \sigma_{\psi} > S_{\psi, i} \)), then the distance of agent \( i \) to its neighbors increases, and decreases otherwise. Choosing a large value of \( \sigma_{\psi} \) will cause small changes in the formation structure, which might not be enough for coping with the noise. On the other hand, choosing a large value will cause an inefficiently large formation at the maximum. Thus, we propose an adaptive noise evaluation method. Let \( \delta_i = |S_{\psi, i}(t)| \), then
\[ \sigma_{\psi, i}(t) = \begin{cases} (\sigma_{\psi, i}(t_{k-1}) + \delta_i)/2, & \sigma_{\psi, i}(t_{k-1}) < \delta_i, \\ \sigma_{\psi, i}(t_{k-1}), & \text{else} \end{cases} \]
where \( \sigma_{\psi, i}(0) = 0 \). The value of \( \sigma_{\psi, i} \) is increasing with respect to the signal’s highest value but does not exceed it. This way we assure that the agents will increase their formation size while seeking the source and reduce it when getting near it. For achieving a smooth behavior of \( \Delta r_f \), a second-order filter \( \frac{\omega_2^2}{s^2 + 2\omega_1 s + \omega_0} \) is included in the output of (31). Let \( \Delta r_f = [\Delta r_{f_1, f_2}, \ldots, \Delta r_{f, N}]^T \) then the formation structure is adapted according to
\[ \Delta r_{fp} = \beta_{r_f} \cdot \Delta r_f \otimes [1, 1]^T, \]
where \( \beta_{r_f} > 0 \) is a scalar tuning gain.

**Proposition 2:** Consider the multi-agent system (1) with control law (7) under the prediction methods for localization (26) and field (32). Under Assumption 1 and assuming that the noise is normally distributed and its power is less than (26) and field (32). Under Assumption 1 and assuming that \( \sigma_{\psi, i} \) then the distance of agent \( i \) to its neighbors increases, and decreases otherwise. Choosing a low value of \( \sigma_{\psi} \) might not be enough for coping with the noise. On the other hand, choosing a large value will cause an inefficiently large formation at the maximum. Thus, we propose an adaptive noise evaluation method. Let \( \delta_i = |S_{\psi, i}(t)| \), then
\[ \sigma_{\psi, i}(t) = \begin{cases} (\sigma_{\psi, i}(t_{k-1}) + \delta_i)/2, & \sigma_{\psi, i}(t_{k-1}) < \delta_i, \\ \sigma_{\psi, i}(t_{k-1}), & \text{else} \end{cases} \]
where \( \sigma_{\psi, i}(0) = 0 \). The value of \( \sigma_{\psi, i} \) is increasing with respect to the signal’s highest value but does not exceed it. This way we assure that the agents will increase their formation size while seeking the source and reduce it when getting near it. For achieving a smooth behavior of \( \Delta r_f \), a second-order filter \( \frac{\omega_2^2}{s^2 + 2\omega_1 s + \omega_0} \) is included in the output of (31). Let \( \Delta r_f = [\Delta r_{f_1, f_2}, \ldots, \Delta r_{f, N}]^T \) then the formation structure is adapted according to
\[ \Delta r_{fp} = \beta_{r_f} \cdot \Delta r_f \otimes [1, 1]^T, \]
where \( \beta_{r_f} > 0 \) is a scalar tuning gain.

**Proof:** Let \( d_1 = ||r_i(t_1) - r_j(t_1)|| \) and \( d_2 = ||r_i(t_2) - r_j(t_2)|| \) where \( d_1 < d_2 \). Let \( \Delta \psi_1 = \Delta \psi_{i,j}(t_1) \) and \( \Delta \psi_2 = \Delta \psi_{i,j}(t_2) \). According to Assumption 1 (single maximum), it follows that \( \Delta \psi_1 < \Delta \psi_2 \). According to (29),
\[ \hat{g}_1 = \frac{\Delta \psi_1}{d_1} - \frac{\Delta n_{\psi_{i,j}}}{d_1}, \quad \hat{g}_2 = \frac{\Delta \psi_1}{d_2} - \frac{\Delta n_{\psi_{i,j}}}{d_2} \]
so we obtain
\[ ||g - \hat{g}_1|| > ||g - \hat{g}_2|| \]
thus the error \( e_g \) monotonically reduces and converges to
\[ \lim_{t \to \infty} e_g = \frac{\Delta n_{\psi_{i,j}}}{d_1}. \]
The agents detect the first obstacle during the source-seeking task and the second obstacle is detected by the eastern patrol group. The agents succeed in avoiding collision during tracking the specified level curve using the anchors as a temporary target.

without any estimation, SNR and a combination of SNR and distributed localization method. Let $\bar{d}$ be the average traveled distance of agents be

$$\bar{d} = \frac{1}{N} \sum_{t_k} ||\bar{r}_x + \bar{r}_y||,$$

(38)

where $\bar{r}_{x,t_k} = \sum_{i} r_{x,t_k}$. This allow us to test the algorithm efficiency where, for the same initial conditions, the shortest distance is desirable. The next parameter is the final error with respect to source location. Due to the formation reference, the minimum error is $e \geq 5$. Let $\bar{e}$ be the Mean Square Error

$$\bar{e} = \frac{1}{N} \sum_{i} ||r_m - r_i||$$

(39)

Table I shows the simulation results. From the results, using

<table>
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<tr>
<th></th>
<th>$\bar{d}$</th>
<th>$\bar{e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>691.6</td>
<td>15.8</td>
</tr>
<tr>
<td>SNR</td>
<td>357.6</td>
<td>15.6</td>
</tr>
<tr>
<td>SNR+Localization</td>
<td>344.7</td>
<td>15.3</td>
</tr>
</tbody>
</table>

**TABLE I: Comparison between different approaches**

SNR reduced $\bar{d}$ significantly, while the combination of SNR and localization provided the best results. Figure 6 shows the agents’ trajectories in both extreme cases without any estimation process and the combined one. It is easy to notice the contribution of the SNR and localization estimation methods to achieving an enhanced trajectory while seeking for the source.

In this paper, an enhanced mission control strategy for exploring time-varying scalar fields in complex environments is proposed. This work is focused on solving field navigation problems in a plane which involve unknown obstacles and noise corrupted measurements. For avoiding obstacles, a virtual potential field is generated based on an adaptive process. A possible upgrade is by extending to a distributed process where the knowledge of agent’s neighbors helps in receiving an accurate image of the obstacle. An additional option is to generate a shared obstacle map for optimized trajectories. A noise compensation scheme is illustrated with simulations, showing the efficiency and robustness of the prediction algorithm.

**REFERENCES**


