3 Sensors

• Displacement and Distance Sensors
• Strain Sensors
• Pressure Sensors
• Temperature Sensors

Displacement and Distance

- Resistive Sensors
- Capacitive Sensors
- Inductive Sensors
- Ultrasonic Systems
- Optical Encoder Systems
- Laser Interferometers
- Triangulation
- Time-of-Flight Systems
Resistive Displacement Sensor

- Electrically conductive wiper that slides against a fixed resistive element according to the position or angle of external shaft
  - Resistance linear function of displacement
  - Calibration maps the output voltage to displacement

Capacitive Displacement Sensors

- Two electrodes:
  - Area $A$
  - Distance $d$
  - Dielectric constant $\varepsilon$

- Capacity

\[ C = \frac{Q}{U} = \varepsilon \frac{A}{d} = \varepsilon_0 \varepsilon_r \frac{A}{d} \]

$\varepsilon_0 = 8.8544 \times 10^{-12} \text{ N}^{-1}\text{m}^2\text{C}^2$: dielectric constant of vacuum
$\varepsilon_r$: relative dielectric constant (relative permittivity)
$Q$: charge on plate of capacitor
Variable Distance Capacitive Displacement Sensor

- Capacity is function of separation of electrodes
  - Nonlinear characteristic
  - Sensitivity:
    \[
    \frac{\partial C}{\partial d} = -\varepsilon_0 \varepsilon_r \frac{A}{d^2}
    \]
  - High sensitivity at small distances (and large Area)
  - Measurement without contact with the object possible
  - Displacements down to atomic scale (motion detectors)

\[\varepsilon_0: \text{dielectric constant of vacuum}\]
\[\varepsilon_r: \text{relative dielectric constant}\]
\[Q: \text{charge on plate of capacitor}\]
Variable Area Capacitive Displacement Sensor

- Varying the surface area of the electrodes of a flat plate capacitor

\[ C = \varepsilon_0 \varepsilon_r \frac{A}{d} = \varepsilon_0 \varepsilon_r \frac{A_0 - wx}{d} \]

\( A_0 \) : Area of the plates
\( w \) : width of the plates

- Linear characteristic
- Displacements in the cm - range
- Used as rotating capacitor for angular measurement
Multiple electrodes

• Varying the surface area of the electrodes of a flat plate capacitor

• Used in (some) vernier calipers
Capacitive Vernier Caliper

• Example 1: Vernier Caliper
  • Two multiplate arrangements, 90° phase shift for determination of direction
  • Typical repeatability: 10 µm

• Example 2:
  • Precision positioning system with 40 µm distance between electrodes
  • Position uncertainty of 4 nm

Variable Dielectric Capacitive Displacement Sensor

- Relative movement of dielectric material between plates

\[ C = \varepsilon_0 w \frac{(\varepsilon_2 (l - x) + \varepsilon_1 x)}{d} = \varepsilon_0 w \frac{(\varepsilon_2 l - (\varepsilon_2 - \varepsilon_1)x)}{d} \]

- \( \varepsilon_1 \) : relative dielectric constant
- \( \varepsilon_2 \) : relative dielectric constant

- Used for level measurements

Differential Capacitive Sensor

- Position of central plate is measured

\[ C_1 = \varepsilon_0 \varepsilon_r \frac{A}{d_0 + x} \]

\[ C_2 = \varepsilon_0 \varepsilon_r \frac{A}{d_0 - x} \]

- Voltage

\[ U = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} U_0 = \frac{1}{2} U_0 \left( 1 + \frac{x}{d_0} \right) \]

- Linear characteristic
Application of Capacitive Sensors

• Linear motions
• Proximity sensing
• Non contact measurements possible
• Well suited for extremely small displacements
• Capacities and capacity differences are small (same order of magnitude as cable capacities) => Preamplification directly at the sensor
• Positive properties: Low power consumption, low temperature dependence, large temperature range, high resolution
• Negative properties: High impedance, small range (variable distance), influenced by dust and moisture
Inductive Displacement Measurement

- **Coil**
  - **Flux density**\[ B = \frac{\mu_0 \mu_r nI}{l} \]
  - **Flux**\[ \phi = \int_B \cdot d\vec{a} = \frac{\mu_0 \mu_r A}{l} \times nI = \frac{1}{R_m} \times nI \]
  - **Reluctance**\[ R_m = \frac{l}{\mu_0 \mu_r A} \]
  - **Self inductance**\[ L = \frac{n\phi}{I} = \frac{1}{R_m} \times n^2 \]
Linear Variable-Reluctance Sensor

Permeability $\mu_A$

Air gap $d$

Central flux path

Permeability $\mu_C$

$R$

$t$
Linear Variable-Reluctance Sensor

- Ferromagnetic core \( \mu_C \)
- Ferromagnetic plate (Armature, \( \mu_A \))
- Air gap \( d \) (\( \mu_{\text{Air}} \approx 1 \))

- Total reluctance
  \[ R_{\text{Total}} = R_C + R_G + R_A \]

- Using
  \[ l_C = \pi R \quad A_C = \pi r^2 \quad R_C = \frac{R}{\mu_0 \mu_C r^2} \]

- \( l_G = d \quad A_G = \pi r^2 \quad R_G = \frac{2d}{\mu_0 \pi r^2} \) \quad (2 gaps, \( \mu_{\text{Air}} \approx 1 \))

- \( l_A \approx 2R \quad A_A \approx 2rt \quad R_A = \frac{R}{\mu_0 \mu_A rt} \quad t: \text{thickness} \)
Linear Variable-Reluctance Sensor

- Calculation of the self inductance:

\[
L = \frac{1}{R_{\text{Total}}} \times n^2
\]

Only one term dependent on \(d\):

\[
R_{\text{Total}} = \frac{R}{\mu_0 \mu_C r^2} + \frac{2d}{\mu_0 \pi r^2} + \frac{R}{\mu_0 \mu_A r t} = R_0 + kd
\]

\[
R_0 = \frac{R}{\mu_0 r} \left( \frac{1}{\mu_C r} + \frac{1}{\mu_A t} \right)
\]

Self inductance as a function of \(d\):

\[
L = \frac{n^2}{R_0 + kd}
\]

- Nonlinear characteristic
- Typical resolution: 0.1 mm
- Proximity sensor
Variable-Differential Reluctance Sensor

- Push-pull sensor
- Linearization
- Bridge Circuit

Linear-Variable Inductor

- Two coils
- Area $A$
- Core:
  - Permeability $\mu_C \gg 1$

- One Coil (Approximation using $\mu_C \gg 1$):
  \[ L_1 = \frac{n^2 \mu_0 A}{h} \]
  - Nonlinear relationship

- Linearization: Differential Setup in Bridge Circuit
- Voltage linear function of displacement for small displacements
## Linear-Variable Inductor

- **Examples:**

<table>
<thead>
<tr>
<th>Type</th>
<th>W1</th>
<th>W20</th>
<th>W200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum displacement</td>
<td>± 1 mm</td>
<td>± 20 mm</td>
<td>± 200 mm</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>4 V/mm</td>
<td>0.2 V/mm</td>
<td>20 mV/mm</td>
</tr>
<tr>
<td>Linearity (Percentage of full scale)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Resolution</td>
<td>10⁻⁶ m</td>
<td>2×10⁻⁵ m</td>
<td>2×10⁻⁴ m</td>
</tr>
<tr>
<td>Frequency (Supply Voltage)</td>
<td>5 kHz</td>
<td>5 kHz</td>
<td>5 kHz</td>
</tr>
<tr>
<td>Supply Voltage</td>
<td>5 V</td>
<td>5 V</td>
<td>5 V</td>
</tr>
<tr>
<td>Temperature Range</td>
<td>-200°C...+150°C</td>
<td>-200°C...+100°C</td>
<td>-200°C...+100°C</td>
</tr>
</tbody>
</table>
Linear Differential Variable Transformer

Coil 1 secondary
Primary coil
Coil 2 secondary
Insulating form
Core

Motion to be indicated or controlled

Difference output voltage

AC supply
Linear Differential Variable Transformer

(a)

\[ V_s \sin 2^{\text{ft}} \]

Primary

\[ x = 1 \]

Secondaries

Phase sensitive demodulator + low pass filter

(b)

Voltage out

Displacement

Core position

Phase

\[ 0 \]

\[ + \]

\[ -180^\circ \]

\[ - \]

\[ + \]

Linear Differential Variable Transformer

- Non-contact device with little friction
- Hysteresis small
- Low susceptibility to interferences
- Solid construction possible
- Temperature ranges: -265 °C to 600 °C
- Typical ranges: ± 0.25 mm to ± 75 mm
- Measurement of displacements in the sub-micrometer range
- Typical linearity: 1% full scale
- Typical Sensitivity 300 mV/mm
Properties of Inductive Displacement Sensors

- Medium displacements (typical mm-cm range)
- Low drift
- High output signal
- High resolution (nm-range possible)
- Low influence of dust
- Negative: Higher forces than capacitive sensors
- Temperature dependence and nonlinearity are compensated by differential setup
Ultrasonic Distance Measurement

- Measurement of “time of flight” of sound

\[ d = \frac{vt}{2} \quad (\Theta \approx 0) \]

- Calculation of the distance \( d \):

- \( v \): velocity of sound
- \( t \): time
Ultrasonic Distance Measurement

• Velocity of sound in a gas

\[ v = \sqrt{\frac{\gamma p}{\rho}} \]

\( \gamma \): ratio of the specific heat at a constant pressure over the specific heat at a constant volume \( \gamma = \frac{c_p}{c_v} \)

\( p \): pressure

\( \rho \): density of the gas

• Velocity of sound in an ideal gas

\[ v = \sqrt{\frac{\gamma RT}{M}} \sim \sqrt{T} \]

\( R \): Gas constant

\( T \): Temperature

\( M \): Molecular weight (kg/mol)

<table>
<thead>
<tr>
<th>Medium</th>
<th>Velocity ( v ) (m/s) at ( p = 1 ) atm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air (0 °C)</td>
<td>331</td>
</tr>
<tr>
<td>Air (20 °C)</td>
<td>343</td>
</tr>
<tr>
<td>Helium (0 °C)</td>
<td>965</td>
</tr>
<tr>
<td>Hydrogen (0 °C)</td>
<td>1284</td>
</tr>
</tbody>
</table>

• Ideal gas: velocity of sound independent of pressure!

• Air: Dependence on pressure negligible in most applications!
Ultrasonic Transducers/Application

Early Application: Autofocus
Ultrasonic Transducer/Application
Car with Ultrasonic Distance Sensors
Ultrasonic Distance Measurement Methods

- Pulse echo method
  - Time-of-flight of a pulse is measured
  - Simple setup
  - Low signal-to-noise ratio
- Phase angle method
  - Phase angle between continuous transmitted signal and continuous received signal is measured
  - Cannot be used directly at distances longer than the wavelength of the ultrasound
- Correlation method
  - Cross-correlation function between transmitted and received signals is calculated
  - Robust against disturbances
Example: Cross-Correlation
Ultrasonic Distance Measurement

Signal sent by ultrasonic transducer

![Signal sent by ultrasonic transducer](image)

Signal received by ultrasonic transducer

![Signal received by ultrasonic transducer](image)

Cross correlation of the two signals

![Cross correlation of the two signals](image)

\[ d = \frac{v t_{\text{flight}}}{2} \quad (\Theta \approx 0) \]
Cross-correlation

- Cross correlation function
  \[ r_{fg}(t) = \int_{-\infty}^{\infty} f^*(\tau) \cdot g(t + \tau) d\tau \]
- Example:
Cross correlation

- Example: Cross correlation function of $f$ and $g$

![Graph showing the cross correlation function with $t = 7.75$ ms]
Cross correlation

- Example: Superposition of the two signals

\[ f(t+\Delta t) \quad \Delta t = 7.75 \text{ ms} \quad g(t) \]
Absolute Direct Binary Encoder

- Number of tracks determined by number of binary digits
  - e.g. 65536 positions $\Rightarrow$ 16 tracks ($2^{16}=65636$)

- Problem: Ambiguity when bits are changing

Gray Code Absolute Encoder

- Gray code: only one bit changes at each transition
Gray Code

Binary code to binary reflected Gray

<table>
<thead>
<tr>
<th>Base_{10} number</th>
<th>“Natural” binary</th>
<th>Gray code</th>
<th>Binary Coded Decimal (BCD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0000</td>
<td>0000</td>
<td>0000 0000</td>
</tr>
<tr>
<td>1</td>
<td>0001</td>
<td>0001</td>
<td>0000 0001</td>
</tr>
<tr>
<td>2</td>
<td>0010</td>
<td>0011</td>
<td>0000 0010</td>
</tr>
<tr>
<td>3</td>
<td>0011</td>
<td>0010</td>
<td>0000 0011</td>
</tr>
<tr>
<td>4</td>
<td>0100</td>
<td>0110</td>
<td>0000 0100</td>
</tr>
<tr>
<td>5</td>
<td>0101</td>
<td>0111</td>
<td>0000 0101</td>
</tr>
<tr>
<td>6</td>
<td>0110</td>
<td>0101</td>
<td>0000 0110</td>
</tr>
<tr>
<td>7</td>
<td>0111</td>
<td>0100</td>
<td>0000 0111</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>1100</td>
<td>0000 1000</td>
</tr>
<tr>
<td>9</td>
<td>1001</td>
<td>1101</td>
<td>0000 1001</td>
</tr>
<tr>
<td>10</td>
<td>1010</td>
<td>1111</td>
<td>0001 0000</td>
</tr>
</tbody>
</table>

Binary reflected Gray code to binary code

NUMERICAL RECIPES IN C
(ISBN 0-521-43108-5)
Conversion

- Calculation of “binary reflected” Gray code from binary code:
  - Perform modulo 2 addition of the bit to be converted and next more significant bit of the binary
    (Practical: XOR the original unsigned binary with a copy of itself that has been right shifted one bit)

- Conversion of “binary reflected” Gray code to binary code:
  1. Start with first Gray Code digit (MSB): binary digit = Gray digit
  2. Add next Gray digit and perform modulo 2 operation to calculate next binary digit
  3. Apply 2. to the result to calculate next bit

- Interpretation of “binary reflected” Gray code:
  - A “1” in the Gray code indicates that the bit of the binary code is different than the previous (more significant) bit
  - A “0” in the Gray code indicates that bit of the binary code is the same as the previous (more significant) bit
Shaft Encoders

- “Natural” Binary Code
- Gray Code
Pseudo Random Binary Code (Sequence)

- Pseudorandom encoding allows the use of only two tracks to produce an absolute encoder.
- A pseudorandom binary sequence (PRBS) is a series of binary records or numbers, generated in such a way that any consecutive series of $n$ digits is unique.

Synchronization track

Pseudo random binary sequence (PRBS)

Detector (e.g. $n$ photo diodes)

Width of one bit: $l$
PRBS

• Chain code: The first \( n - 1 \) digits of an \( n \)-bit word are identical to the last \( n - 1 \) digits of the previous code word
• \( n \) bits: \( 2^n \) different codes
• Total length of the scale:
  \[ L = (2^{n-1}) \times l \]

• Possible calculation:
  1. A code (\( n \) random bits) containing nonzero elements is used
  2. Calculation of next bit using

\[
X_{new}(0) = X(n) \cdot c(n) \oplus X(n-1) \cdot c(n-1) \oplus \ldots X(1) \cdot c(1)
\]

\( X(i) \) : Value of bit \( i \) of the current code, \( c(i) = 0,1 \) (see table), \( \oplus \): modulo 2 addition

<table>
<thead>
<tr>
<th>Number of bits</th>
<th>Total length</th>
<th>Nonzero coefficients ( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>15</td>
<td>1, 4</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
<td>2, 5</td>
</tr>
<tr>
<td>6</td>
<td>63</td>
<td>1, 6</td>
</tr>
<tr>
<td>7</td>
<td>127</td>
<td>3, 7</td>
</tr>
<tr>
<td>8</td>
<td>255</td>
<td>2, 3, 4, 8</td>
</tr>
<tr>
<td>9</td>
<td>511</td>
<td>4, 9</td>
</tr>
<tr>
<td>10</td>
<td>1023</td>
<td>3, 10</td>
</tr>
<tr>
<td>11</td>
<td>2047</td>
<td>2, 11</td>
</tr>
<tr>
<td>12</td>
<td>4095</td>
<td>1, 4, 6, 12</td>
</tr>
<tr>
<td>13</td>
<td>8191</td>
<td>1, 3, 4, 13</td>
</tr>
<tr>
<td>14</td>
<td>16383</td>
<td>1, 6, 10, 14</td>
</tr>
</tbody>
</table>
Interferometric Displacement Measurement

- Michelson Interferometer

![Diagram of a Michelson Interferometer](image)
Interference

- Superposition of electromagnetic waves
  - Amplitude of Superposition of
    \[ E_1 = \alpha e^{i(\omega t - ky)} \]
    \[ E_2 = \alpha e^{i(\omega t - ky + \phi)} \]
    is \[ A = \alpha + \alpha e^{i\phi} \]

![Waveforms with different phase differences]

- Phase difference \( \phi = 0 \)
- Phase difference \( \phi = \frac{\pi}{2} \)
- Phase difference \( \phi = \pi \)
Interferometric Displacement Measurement

- Michelson Interferometer
Michelson Interferometer

- Plane electromagnetic wave

  Angular Frequency: $\omega_L = 2\pi c \sigma$

  Wavenumber: $\sigma = 1/\lambda = f/c$  ($k = 2\pi/\lambda$)

  Frequency $f$, Wavelength $\lambda$ (vacuum)

  Velocity of light (vacuum): $c$

  $E(z,t) = A_0 e^{i(\omega_L t - kz)}$

  $A_0$: Amplitude (complex)

- Beamsplitter divides beam into two coherent beams

- After reflection at the mirrors 1 and 2, the beams are recombined with an optical path difference $x = 2d$ (paraxial rays)

- Phase Difference: $\varphi = 2\pi \sigma x = 2\pi \sigma (2d) = 4\pi \sigma d$ (Real Beamsplitter: Additional Phase shift $\varphi_0 = \pi$ for external reflection)
Calculation of the Irradiance at the Detector

- Amplitude of Superposition of

\[ E_1(y, t) = \alpha e^{i(\omega_L t - ky)} \quad E_2(y, t) = \alpha e^{i(\omega_L t - ky + \varphi)} \]

is

\[ A = \alpha + \alpha e^{i\varphi} \]

- Irradiance \( I \) (W/m\(^2\)) is proportional to the square of the Amplitude

\[ I_\sigma \sim A A^* \]

\[ I_\sigma(\varphi) \sim \alpha^2 + \alpha^2 + \alpha^2(e^{i\varphi} + e^{-i\varphi}) \]

\[ I_\sigma(\varphi) \sim 2\alpha^2 + 2\alpha^2 \cos \varphi = 2\alpha^2(1 + \cos \varphi) \]

- \( \bar{I} \): mean value of \( I \)

\[ I_\sigma(x) = \bar{I}(1 + \cos 2\pi\sigma x) = \bar{I}(1 + \cos 4\pi\sigma d) \]
Michelson-Interferometer: 
Monochromatic Radiation

\[ S_{\sigma_1} \]

Beamsplitter

Detector

Signal: 

\[ \lambda_1 = 1/\sigma_1 \]

\[ \frac{l}{l_{\text{max}}} \]

Optical path difference / \( \lambda_1 \)

Distance \( d / \lambda_1 \)

Phase difference \( \varphi = 0 \)

Phase difference \( \varphi = \pi \)
Two-frequency heterodyne interferometer

Laser source $(f_1$ and $f_2$)

Orthogonal polarized modes

Beam-splitter

(1) $f_1$

(2) $f_2$

(3) $f_1 \pm f_D$

(4) $f_2$

(5) $f_1 \pm f_D$

Polarizers rotated 45°

$-f_2-f_1$

$-f_2-(f_1 \pm f_D)$

Photo detectors

Reference counter

Counter

Difference

Reflectors

Two-frequency heterodyne interferometer

- **Source**: Stabilized laser, two frequencies $f_1, f_2$
- **Frequency difference**: $f_2 - f_1 \approx 1 - 20$ MHz
- **Doppler frequency** $f_D$

$$f_D = \frac{2}{\lambda} v$$

- $\lambda$: Wavelength ($f_1$)
- $v$: Velocity

- **Calculation of the difference of the counters yields displacement in multiples of $\lambda/2$**

- **Evaluation of the phase $\Rightarrow$ higher resolution**
Two-frequency heterodyne interferometer

- Example: Mirror moving away from beam splitter
  - After time $t_0$ a difference of 1 count is detected:
  - Counter 1: $n = (f_2 - f_1) t_0$  
  - Counter 2: $n + 1 = (f_2 - (f_1 - f_D)) t_0$
  - Calculation of displacement $x$:
    
    Counter 2: $n + 1 = (f_2 - f_1) t_0 + f_D t_0$
    
    $n + 1 = n + f_D t_0$  
    $1 = f_D t_0$  
    $f_D = \frac{2}{\lambda} v$
    
    $1 = \left( \frac{2}{\lambda} v \right) t_0 = \left( \frac{2}{\lambda} \frac{x}{t_0} \right) t_0$
    
    $x = \frac{\lambda}{2}$
Interferometric Displacement Measurement (General)

- Extremely small relative uncertainty possible \((5 \times 10^{-8})\)

- Uncertainty of refractive index of air ultimately limits performance

- Commercially available: Displacements down to \(\lambda/512\) \((\lambda \approx 633\text{ nm})\)

- Other Interferometers for displacement measurements: Mach-Zehnder Interferometer, Fabry-Perot-Interferometer
Interferometric Distance Measurement / Definition

- 1960-1983: Definition of the meter using wavelength of the orange line of $^{86}\text{Kr}$
- Realised by Michelson-Interferometer (until today)

1892–93: Michelson Interferometer at BIPM in Paris
Interferometric Positioning System

- Semiconductor Inspection
- X-Ray lithography: Positioning
Interferometric Positioning System with Column Referencing
Application of Interferometric Displacement Measurement

- Servo-Track Writing (hard drives and optical storage systems)
Hard Disk

http://www.duxcw.com
Multiaxis System for a Coordinate Measuring Machine
Triangulation

- Principle: Triangle may be constructed by length of one side and two angles

\[ R = \frac{b \sin \alpha_{left} \sin \alpha_{right}}{\sin(\alpha_{right} - \alpha_{left})} \]
Triangulation for Proximity Sensing

- Theorem of intercepting lines
  \[
  \frac{x}{y_0} = \frac{x + h}{y_i}
  \]
  \[
  x = \frac{y_0 h}{y_i - y_0}
  \]

- Receivers: For example CCD
Industrial Application of Triangulation Systems

- Range: Small to medium distances (μm – cm)
- Uncertainty dependent on range (example: 1 - 1500 μm)

- Positioning of robots (here: assembly of lights of a car)
Time-of-Flight Distance Measurement

- Measurement of the time-of-flight of light:  \[ d = \frac{ct}{2} \]
- \( c \approx 3 \times 10^8 \text{ m/s} \Rightarrow \text{measurement of time in the ps} - \text{range required for uncertainty in the mm} - \text{range} \)

http://www.acam.de/
Time-of-Flight Distance Measurement

- **Methods:** 1. Pulse method, 2. Phase method (cross correlation)
- **Hand-held devices**
  - Typical range: 0.2 – 200 m
  - Uncertainty: 1.5 mm (whole range)
- **Industrial systems**
  - Medium distances (typically m-range)
  - Positioning of stackers
  - Positioning of robots
  - Proximity switches
  - Safety applications
  - Self guided vehicles (laser scanners)
  - Future application: safety systems for cars
Laser Scanners (Distance Measurement)

- Classification of luggage
- Control of robot

www.sick.de
Distance Measurement in an Assembly Line

- Distances in an assembly line
- Time-of-Flight (correlation method)
Road Surveillance

- Automated (autonomous) truck
- Laser scanner detects obstacles
Application of Laser Scanners
Autonomous Car
Uncertainty as a Function of Distance for Triangulation and Time-of-Flight Methods

- **Time-of-Flight**: Uncertainty given by uncertainty of measurement of time ⇒ constant
- **Triangulation**: Propagation of uncertainty

\[ \sigma_{\text{Tria}} = \frac{x^2}{y_0h} \sigma y_i \sim x^2 \]
**Strain Gages**

- Resistor bonded with an elastic carrier

\[
R = \frac{\rho l}{A}
\]

- A : area of the cross section
- \(\rho\) : resistivity
- \(l\) : length

- Relative resistance change

\[
\frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta l}{l} - \frac{\Delta A}{A} = \left(1 + 2\mu + \frac{\Delta \rho}{\rho} \right) \varepsilon = k \varepsilon
\]

\[
\varepsilon = \frac{\Delta l}{l}, \quad \mu = - \frac{\Delta d/d}{\Delta l/l} \quad k : \text{gauge factor}
\]
# Strain Gages

## Materials:
- **Konstantan** (60 % Cu, 40 % Ni)  
  \[ k = 2 \text{ bis } 300 \, ^\circ\text{C} \]  
  \[ \rho = 0.49 \, \Omega \text{ mm}^2/\text{m} \]

- **Karma** (74 % Ni, 20 % Cr, 3 % Fe, 3 % Al)  
  \[ k = 2 \]  
  \[ \rho = 1.6 \, \Omega \text{ mm}^2/\text{m} \]

- **PtIr**  
  \[ k = 6 \]

- **Semiconductors**  
  (strong temperature dependence)  
  \[ k = -100 \text{ } - 150 \]
Strain Gage

- Construction of a wire strain gage

Fabry-Perot Fiber Optic Strain Gage

1.3 µm light source:

Pressure Sensors

- Strain gage on diaphragm
- Modern pressure sensors with strain gages:
  - Integrated semiconductor devices with semiconductor strain gages
  - Temperature compensated (bridge circuits)
Capacitive Pressure Sensor

- Distance between electrodes is a function of pressure (difference)

Conductive Diaphragm
Metalization

Ceramic or Glass Substrate
Through-Hole

Equivalent Circuit

$P_1$

$P_2$
Capacitive Pressure Sensor

- Distance between electrodes is a function of pressure difference